

RESTORATION UNIQUE FACTORIZATION USING ROOTS OF QUARTIC POLYNOMIALS

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Unique Factorization Domains (UFD) are important mathematical structures that arise in all areas of abstract algebra. An integral domain is called a Unique Factorization Domain if every non-zero non-unit element can be written as a product of irreducible elements uniquely up to order and multiplication by units. In this research, the concepts of loss and restoration of the Unique Factorization property were explored. For example, \mathbb{Z} is a unique factorization domain, however, if the element $\sqrt{-3}$ is used in extending the ring to $\mathbb{Z}[\sqrt{-3}]$, the Unique Factorization property is lost. Unique Factorization can be restored by extending $\mathbb{Z}[\sqrt{-3}]$ by $\left[\frac{1+\sqrt{-3}}{2}\right]$. Restoring Unique Factorization is subject to extensive research as a general method of doing restoration is yet to be found. In this project, it is looked in to restoring Unique Factorization by extending the integral domain using a radical, which is a root of a polynomial of a certain degree, called the degree of Unique Factorization Restoration Extension (UFRE). Literature reports indicate that all UFREs of degree 2 have been classified for an extension by a radical of the form $\sqrt{-a} : a > 0$. In this research, it is explicitly shown that the loss of unique factorization of $\mathbb{Z}[\sqrt{-5}]$ cannot be restored by a UFRE of degree 2; but using an argument based on Minkowski's estimate, a UFRE of degree 4 can be found.

Keywords: Minkowski's estimate, Restoration of unique factorization.