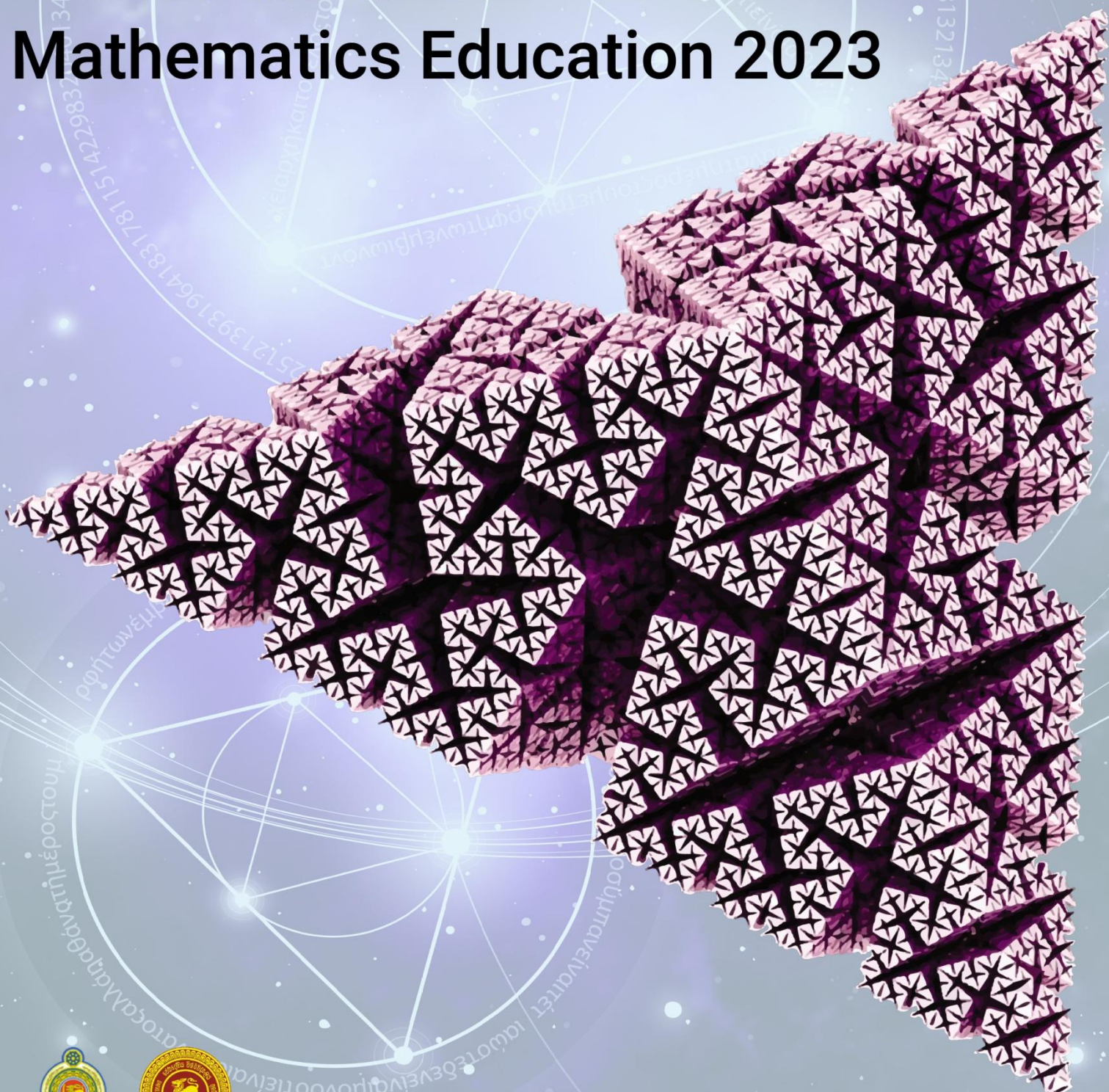


Proceedings International Conference on Mathematics and Mathematics Education 2023



**POSTGRADUATE INSTITUTE OF SCIENCE
UNIVERSITY OF PERADENIYA
SRI LANKA**



PROCEEDINGS

**INTERNATIONAL CONFERENCE ON
MATHEMATICS AND
MATHEMATICS EDUCATION**

14th and 15th of July, 2023

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TABLE OF CONTENTS

Message from the Director, Postgraduate Institute of Science	vii
Message from the Chairperson	ix
Message from the Chairperson/Board of Study in Mathematics	xi
Message from the Chairperson/Board of Study in Science Education	xiii
A Brief Biography of Professor Asha Rao	xv
The Abstract of the Keynote Speech of Professor Asha Rao	xvii
A Brief Biography of Professor Dinesh Ekanayake	xix
The Abstract of the Keynote Speech of Professor Dinesh Ekanayake	xxi
A Brief Biography of Professor Berinderjeet Kaur	xxiii
The Abstract of the Keynote Speech of Professor Berinderjeet Kaur	xxv
Editorial board	xxvii
List of Abstracts.....	xxix

Abstracts

Theme - Mathematics	1
Theme - Mathematics Education	43
Organizing Committee.....	61
Sponsors	63
Author Index	66

ICMME 2023

Message from the Director, Postgraduate Institute of Science



With immense delight, I am pleased to convey this message in celebration of the successful conclusion of the 3rd International Conference on Mathematics and Mathematics Education (ICMME-2023). The esteemed event took place on the 14th and 15th of July, 2023, hosted at the Postgraduate Institute of Science (PGIS), University of Peradeniya.

The ICMME-2023 was organized with the collaboration of the Ministry of Education in Sri Lanka. Mathematics is a fundamental part of our daily lives, and it helps us do many things that are important in our everyday lives. Understanding mathematics is crucial for individuals in all fields across the globe. Numerical concepts and arithmetic operations are integral to every facet of life. Mathematics serves as the fundamental language underpinning the entire realm of science and technology. It not only enables the advancement of scientific and technological disciplines but also empowers us to grasp patterns, forge connections, and predict outcomes. The significance of mathematics extends to a wide spectrum of essential daily tasks. Elevating the standard of mathematics education in schools is a primary goal of this conference.

The PGIS organizes national and international conferences, workshops and short courses. The ICMME provides a platform for postgraduate researchers and educators to present their findings engage in discussions, and exchange insights. Anticipating the unification of mathematics educators nationwide, this conference will create a forum and foundation for collaborative endeavors. The conference will cover various themes, each with unique social dimensions.

The successful realization of this conference is attributed to the unwavering dedication and relentless efforts of our esteemed colleagues. Our heartfelt appreciation goes out to Dr. RajithaRanasinghe, the Conference Chair, and Dr. ChathurikaAthapattu, the Secretary of ICMME, for their meticulous planning and exceptional leadership that culminated in this outstanding event. Gratitude is also extended to the members of the organizing committee and the referees, whose invaluable contributions significantly enriched the review process. We extend our sincere thanks to all the authors who entrusted their work to this conference, and extend our heartfelt congratulations to all the presenters who will be sharing their insightful findings during the conference.

Prof. H.M.T.G.A. Pitawala

Director

Postgraduate Institute of Science

University of Peradeniya, Peradeniya

Sri Lanka

ICMME 2023

Message from the Chairperson



It is with immense pleasure and honor that I welcome you all to the proceedings of ICMME 2023, held on 14th and 15th of July 2023 at the Postgraduate Institute of Science, University of Peradeniya. This conference stands as a testament to the dedication, innovation, and collaborative spirit that define the academic and professional community of mathematics and mathematics education.

The success of this conference would not have been possible without the hard work and commitment of numerous individuals. I extend my heartfelt gratitude for the unwavering support and guidance extended by Prof. H.M.T.G.A. Pitawala, Director/PGIS, without whom the conference would not have seen the daylight. I'm indebted to the untiring efforts of the Secretary of ICMME 2023, Dr. A.M.C.U.M. Athapattu for keeping track of each and every task and for making sure things are in order. A special mention to Dr. U.D. Wijesooriya for taking care of the tasks of the Editorial Committee. My heartfelt thanks go out to the conveners and members of the sub-committees for generously devoting their time and expertise to uphold the standards and make the conference a reality. Your contributions are truly appreciated.

Equally deserving of recognition are the authors whose research findings grace these proceedings. Your dedication to advancing knowledge and pushing the boundaries of our understanding is truly inspiring. It is my hope that the discussions held during the conference will continue to reverberate through your work and inspire future collaborations.

At a time in which financial constraints hamper organizing events of this nature, for providing us invaluable financial support, I extend my heartfelt appreciation to Mr. D.M.S.D. Dissanayake (Department of Education, Central Province) and Mrs. S.S. Dodanwala (Manager, Bank of Ceylon, Peradeniya University Branch).

Also, my sincere gratitude to Prof. Asha Rao, the keynote speaker, as well as Prof. Dinesh Ekanayake and Prof. Berinderjeet Kaur, our distinguished plenary speakers in mathematics and mathematics education, respectively, for enriching the event with their enlightening and engaging presentations.

Let these proceedings serve as a testament to our collective commitment to driving progress and making a lasting impact. I encourage you all to immerse yourselves in the rich content presented here, engage in thoughtful conversations, and forge connections that will propel our community forward.

Thank you and best wishes!

Dr. P.G.R.S. Ranasinghe
Chairperson/ICMME 2023

ICMME 2023

Message from the Chairperson/Board of Study in Mathematics



It is with great pleasure that I send this message to the Third International Conference on Mathematics and Mathematics Education (ICMME) 2023 organized by the Postgraduate Institute of Science, University of Peradeniya in collaboration with the Ministry of Education.

Marking its third milestone, with the theme 'Mathematics for a Better World', ICMME aims to disseminate mathematics knowledge and provide a platform to showcase the research findings in mathematics carried out by the researchers in Mathematics and Mathematics Education around the world. Further, as a biennial event, it provides a great opportunity for the Sri Lankan researchers and Educationists in Mathematics to share their experience, thoughts and collaborate with the worldwide experts in Mathematics and Mathematics Education.

The Board of Study in Mathematics (BoSM) at the Postgraduate Institute of Science attempts to encourage and cater the needs of the academics and the postgraduate students in conducting research on Mathematics and Mathematics Education and publishing their findings in both National and International Journals. In collaboration with the Board of study in Science Education, in 2019, BoSM initiated organizing the ICMME to bring academics in Mathematics and Mathematics Educationists to one common platform. I am confident that ICMME continues to serve one of the main conferences in Sri Lanka.

Finally, I would like to take this opportunity to express my sincere gratitude to Prof. H.M.T.G.A. Pitawala, the Director/PGIS, Dr. P.G.R.S. Ranasinghe, the Chairperson of ICMME 2023, the Members of the Organizing Committee, and the members of the Boards of Studies in Mathematics and Science Education for their dedication and hard work to make this event a success. Further, I would like to congratulate all the presenters and wish great success for the conference.

Dr. T.H.K.R. De Silva

*Chairperson/Board of Study in Mathematics
Postgraduate Institute of Science,
Sri Lanka*

ICMME 2023

Message from the Chairperson/Board of Study in Science Education



As the Chairperson of the Board of Study in Science Education, it is my privilege to provide a message to mark the Third International Conference of Mathematics and Mathematics Education (ICMME) 2023 to be held on 14th and 15th July, 2023 at the Postgraduate Institute of Science, University of Peradeniya.

Literacy in mathematics is crucial for the economic progress of countries and has shown a positive relationship between students' performance in mathematics and economic growth. Proficiency in mathematics is also indicative of cognitive skills among the work force of a country, thus enhancing the quality of human resources, which will eventually lead to technological innovations and productivity. A solid foundation in mathematics education not only produces engineers and scientists, but also produces citizens who can learn and think creatively and critically, which is a must in today's workforce.

Recognizing this national obligation, the Postgraduate Institute of Science, University of Peradeniya offers postgraduate degree programs in Science Education in which Mathematics Education includes as a major discipline. In addition, the PGIS rightly provides a platform for those who are interested in research in the fields of mathematics and mathematics education by organizing an International Conference on Mathematics and Mathematics Education (ICMME) in 2019. Since then, this conference has become a focal point for researchers and educators alike to present and share their findings as well as to build-up fruitful collaborations.

It is indeed a pleasure to be part of this energetic young team who organizes this event for the third time amidst the country's economic downturn. I would like to take this opportunity to thank the Organizing Committee headed by Dr. P.G.R.S. Ranasinghe, Chairperson of ICMME 2023 for making this event a success. Finally, I would like to wish all participants a rewarding academic experience!

Prof. H.M.S.P. Madawala

Co-Editor-In-Chief – ICMME 2023

Chairperson/Board of Study in Science Education

Postgraduate Institute of Science

Sri Lanka

ICMME 2023

A Brief Biography of Professor Asha Rao



Professor Asha Rao is an accomplished academic and currently serves as the Associate Dean of Mathematical Sciences at the RMIT University. Her diverse research interests span communication technologies, social media, and architecture, and her trans-disciplinary research approach has resulted in a strong publication record and the acquisition of over \$1.5M in funding over the past 8 years. Professor Rao's expertise in risk, particularly in the areas of fraud and money laundering, has earned her invitations to participate in various national and international bodies, notably, including the 4th United Nations Intergovernmental Meeting on Cybercrime in early 2018.

In addition to her research, Prof. Rao has demonstrated strong leadership both within and outside RMIT. As the Program Director for Information Security, she established vital connections between the technical degree and the cybersecurity industry, enhancing the relevance of the program. Rao is also the founding chair of Women in Maths at RMIT, implementing several initiatives aimed at improving gender equity within the mathematical sciences.

ICMME 2023

The Abstract of the Keynote Speech of Professor Asha Rao

Identifying a Criminal's Network of Trust

Tracing criminal ties and mining evidence from a large network to begin a crime case analysis has been difficult for criminal investigators due to large numbers of nodes and their complex relationships. I will first describe the crime of money laundering and why it is a problem for society. I will detail some of the work being done by the UN to tackle this crime. I will then talk about a specific research paper. In this paper, trust networks using blind carbon copy (BCC) emails were formed. My co-authors and I showed that our new shortest paths network search algorithm combining shortest paths and network centrality measures can isolate and identify criminals' connections within a trust network. A group of BCC emails out of 1,887,305 Enron email transactions were isolated for this purpose. The algorithm uses two central nodes, most influential and middle man, to extract a shortest paths trust network.

ICMME 2023

A Brief Biography of Professor Dinesh Ekanayake



Professor Dinesh Ekanayake is a Professor and Graduate Coordinator in the Department of Mathematics and Philosophy at Western Illinois University. He obtained a Ph.D. and M.S. in Applied Mathematics from Texas Tech University and a B.S. in Electrical Engineering from the University of Peradeniya in Sri Lanka. Prof. Ekanayake's research focuses on applying advances in control methods to complex hysteretic systems for engineering and biological applications, particularly in the areas of hysteresis modeling, control of hysteretic systems, and wildlife disease dynamics and control. He has published papers in renowned scientific journals such as the Journal of Biological Dynamics, SIAM Journal on Control and Optimization, and IEEE ACC, contributing to the development of proportional derivative control for hysteretic systems and the stabilization of a system with non-monotone hysteresis and frequency-dependent power losses by a PD controller.

ICMME 2023

The Abstract of the Keynote Speech of Professor Dinesh Ekanayake

Mathematics research in multidisciplinary fields

The contributions of mathematics are indispensable in modern multidisciplinary fields that drive the scientific and technological progress of contemporary society, including data science, computational science, atmospheric science, energy systems engineering, and computational biology, among others. In this talk, I will share some research to illustrate the importance and impact of mathematics in confronting intriguing problems in such fields. The presentation will include some very specific research questions such as: what can birds tell us about their nesting roost preferences, and how does the average annual minimum winter temperature impact deer fecundity? The presentation will also include some more general research problems, such as how history be incorporated in disease transmission models, what facet reflections bring to large scale and complex optimization problems, and how tight partitions produce optimal circle packings. I will also discuss the critical need for mathematics education that includes research and applications, even at the high school level, to prepare students for success in multidisciplinary fields.

ICMME 2023

A Brief Biography of Professor Berinderjeet Kaur



Professor Berinderjeet Kaur is a distinguished Professor of Mathematics Education at the National Institute of Education (NIE), Nanyang Technological University (NTU). With a Ph.D. in Mathematics Education from Monash University and over three decades of association with NIE, Kaur has established herself as a prolific researcher, securing more than S\$6.0 million in grants and publishing extensively in national and international journals, books, and conference proceedings. Kaur is an associate editor of *ZDM-Mathematics Education* and a series editor of Springer's *Mathematics Education - An Asian Perspective*. She has received the Excellence in Teaching Commendation four times from NIE and is actively involved in pre-service and in-service education of mathematics teachers at NIE. As a distinguished scholar in Mathematics Education, she served as the Mathematics Consultant to the Trends in International Mathematics and Science Study (TIMSS) in 2011 and the Mathematics Expert for the Programme for International Student Assessment (PISA) in 2015. She is the founding chair of the annual Mathematics Teachers Conference organized by the Association of Mathematics Educators (AME) since 2005. Also, she is the founding editor of the Yearbook of AME published annually by the World Scientific since 2009.

ICMME 2023

The Abstract of the Keynote Speech of Professor Berinderjeet Kaur

Professional Development of Mathematics Teachers and Sustainable Innovation in Classroom Practice

As the proverb goes ‘Give a man a fish and he will eat for a day. Teach a man how to fish and you feed him for a lifetime’ this presentation centers around how teachers may be engaged in professional development that facilitates sustainable innovation in their classroom practice. We draw on work done with teachers in Singapore schools and illustrate how professional development that is school-based, demand oriented, on-line and in situ has sustained innovation in classroom practice. In particular, the hybrid model of professional development comprising five critical features, namely content focus, coherence, duration, active learning and collective participation drives the continuous professional development of the teachers.

ICMME 2023

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ICMME 2023

List of Abstracts

THEME - Mathematics

Abstract No.	Title and Authors	Page No.
01	GRACEFUL LABELLING OF ROTATED IRREGULAR CYCLIC SNAKE GRAPHS <i>W.K.M. Indunil, K.N. Kaluarachchi, S.M.T. Ruwan and A.A.I. Perera</i>	1
02	TUNING THE PARAMETERS AND SELECTING THE OPERATORS FOR THE GENETIC ALGORITHM TO SOLVE THE VEHICLE ROUTING PROBLEM <i>S.R. Gnanapragasam and W.B. Daundasekera</i>	2
03	STUDY OF NITROGEN DIOXIDE ANNUAL AVERAGE CONCENTRATION IN COLOMBO USING INVERSE DISTANCE WEIGHTED AND LOCAL RADIAL BASIS FUNCTION METHODS <i>W.H.D.T. Karunarathna, K.M.N. Dissanayake and M.T.M. Dewasurendra</i>	3
04	MATHEMATICAL MODELLING FOR FINGERO-IMBIBITION DURING CARBONATED WATER FLOODING WITH MAGNETIC FIELD EFFECT <i>C.W. Sahabandu and M.T.M. Dewasurendra</i>	4
05	MDDIM SOLUTIONS IN SOLVING NONLINEAR FUZZY HEAT-LIKE EQUATIONS <i>C.W. Sahabandu and M.T.M. Dewasurendra</i>	5
06	A MATHEMATICAL APPROACH TO FINDING THE OPTIMAL WORKING ROLE ALLOCATION WHICH MINIMIZES THE TOTAL WORKING TIME <i>P.L.M.W.T.P. Jayasinghe and M.S. Wickramarachchi</i>	6
07	FORECASTING AND EXAMINATION OF THE DYNAMIC BEHAVIOUR OF THE GLOBAL FINANCIAL DEVELOPMENT INDEX <i>R.R.W.G.B.M.K.B. Hulangamuwa, R.B.N. Dissanayake and P.K. Premachandra</i>	7
09	ELETING A SLIT FROM THE UNIT DISC <i>M. Arunmaran</i>	8

THEME - Mathematics

Abstract No.	Title and Authors	Page No.
11	A FRACTIONAL ORDER MATHEMATICAL MODEL FOR THE SPREAD OF COVID-19 WITH QUARANTINE AND ISOLATION CONTROL STRATEGIES <i>S.K.D.T.D.Korala and L.W.Somathilake</i>	9
12	ODD PRIME LABELLING OF DOUBLE SNAKE GRAPH <i>K.H.C. De Silva and A.A.I. Perera</i>	10
13	AN ALGORITHM FOR RAINBOW COLOURING OF THE HIGHER ORDER EXTENSION OF THE SANDAT GRAPH <i>G.W.M.M.K. Dheerasinghe, A.A.I. Perera, P.G.R.S. Ranasinghe and K.D.E. Dhananjaya</i>	11
15	SOME TOPOLOGICAL INDICES OF FIRECRACKER GRAPHS <i>J.A.D.C.M. Alwis and K.K.K.R. Perera</i>	12
16	THE ANT COLONY OPTIMIZATION ALGORITHM TO SOLVE THE TRAPEZOIDAL TRANSPORTATION PROBLEMS <i>E.M.T.D.K. Ekanayake and E.M.U.S.B. Ekanayake</i>	13
17	RADIO ARITHMETIC MEAN LABELLING OF PENDANT GRAPHS <i>G.H.S.N. Weerasinghe and A.A.I. Perera</i>	14
18	A LOCAL SEARCH-BASED HYBRID GENETIC ALGORITHM FOR THE VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICKUP AND DELIVERY <i>P.V.V. Viduranga and M.K.D.D. Sandaruwan</i>	15
19	IMMUNITY WANING DYNAMICS OF COVID-19 THROUGH A MATHEMATICAL MODEL <i>B.A.P.N. Sandaruwini and L.W. Somathilake</i>	16
20	AN INTEGER LINEAR PROGRAMMING MODEL TO OPTIMIZE NURSE SCHEDULING: A CASE STUDY <i>A.A.D.H. Ishara and W.B. Daundasekera</i>	17
21	NEW EXPLICIT FORMULAS FOR HIGHER-ORDER APPROXIMATIONS OF FIRST AND SECOND DERIVATIVES WITH APPLICATIONS <i>J.A.M.P. Weerasinghe, W.A. Gunarathna and M.A.M. Mohommad</i>	18
22	CHROMATIC POLYNOMIAL OF A SPECIAL TYPES OF LADDER GRAPH <i>E.H.W. Weerabaddana and A.A.I. Perera</i>	19

THEME - Mathematics

Abstract No.	Title and Authors	Page No.
23	A NOVEL SECOND-ORDER FINITE DIFFERENCE APPROXIMATION FOR FISHER EQUATION <i>C.T. Fernandopulle, W.A. Gunarathna and M.A.M. Mohamed</i>	20
24	SOME PROPERTIES OF FUZZY METRIC SPACES <i>N. Kajan and K. Kannan</i>	21
25	A DATA-DRIVEN APPROACH FOR SIGNATURE VERIFICATION <i>H.D.A.W. Subawickrama, S.A.A. Nishantha, M.A.M. Mohamed and U.G.I.G.K. Udagedara</i>	22
27	PORTFOLIO REBALANCING IN THE PRESENCE OF TRANSACTION COSTS <i>M.W.M.L. Vidumali, S.A.A. Nishantha and U.G.I.G.K. Udagedara</i>	23
28	THE EXTENDED AUXILIARY EQUATION METHOD FOR THE NONLINEAR SCHRÖDINGER EQUATION WITH VARIABLE - COEFFICIENT <i>D. Thavarajah, T. Mathanaranjan</i>	24
29	A NEW FOURTH-ORDER CRANK-NICOLSON DIFFERENCE APPROXIMATION FOR THE DIFFUSION EQUATION <i>S.S. Nawaratne, W.A. Gunarathne and M.A.M. Mohamed</i>	25
30	A TWO-SEX AGE-STRUCTURED POPULATION DYNAMICS MODEL FOR THALASSEMIA TRANSMISSION <i>A.M.P. Chandrasiri and T.H.K.R. De Silva</i>	26
31	ARTIFICIAL BEE COLONY ALGORITHM TO SOLVE LINEAR FRACTIONAL TRANSPORTATION PROBLEM WITH VARYING DEMAND AND SUPPLY <i>R. Gunarathne and W.N.P. Rodrigo</i>	27
32	COMPUTING MULTI-DIMENSIONAL PERSISTENT HOMOLOGY TO STUDY PROTEIN STRUCTURES <i>L.R.M.K.R. Jayathilaka and A.K. Amarasinghe</i>	28
33	AN ALTERNATIVE APPROACH TO PROFIT MAXIMIZATION USING THE TRIANGULAR FUZZY TRANSPORTATION PROBLEM <i>E.M.D.B. Ekanayake and E.M.U.S.B. Ekanayake</i>	29
34	PATTERN GENERATION FOR TWO-DIMENSIONAL CUTTING STOCK PROBLEM <i>S.M. Infas and W.N.P. Rodrigo</i>	30
35	ODD HARMONIOUS LABELLING OF LADDER GRAPHS <i>K.M.P.G.S.C. Kapuhennayake, M.M.C Dilshan and A.A.I Perera</i>	31

THEME - Mathematics

Abstract No.	Title and Authors	Page No.
36	A MANUFACTURES AND BUYERS INTEGRATED PRODUCTION INVENTORY SUPPLY CHAIN MODEL WITH UNEQUAL SIZE OF BATCH TRANSFER <i>M.S.M. Hisam, W.B. Daundasekera, and W.N.P. Rodrigo</i>	32
37	MODELING THE TEA PRICES AT THE SRI LANKA TEA AUCTION: A TIME SERIES APPROACH <i>P. Selvarajah</i>	33
39	UNBALANCED MAXIMIZATION ASSIGNMENT PROBLEM WITH MULTIPLE ASSIGNMENTS <i>C.R. Welgama and W.N.P. Rodrigo</i>	34
44	IMPROVING THE PRODUCTION EFFICIENCY IN APPAREL INDUSTRY THROUGH STATISTICAL PROCESS CONTROL: A CASE STUDY <i>S.T.R.D. Thalwaththe and W.B. Daundasekera</i>	35
45	SKOLEM AND HOOKED SKOLEM SEQUENCES UPTO ORDER EIGHT <i>A.M.S.L.K. Athapaththu and A.M.C.U.M. Athapattu</i>	36
49	ALEXANDROV UNIFORMITY OF INVERSE SEMIGROUPS <i>M.W.S. Randunu, K.M.N.M.Chathurangaand A.M. C.U.M. Athapattu</i>	37
51	OPERATOR NORM OF FUNCTIONALS ON SUBSPACES OF HARDY SPACE OF THE UNIT DISK ($H^2(D)$) AND CONSEQUENCES <i>H.R.H.S. Waidyarathne and D. Thilakarathna</i>	38
52	HOMOTOPY ANALYSIS METHOD FOR NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS OCCURRING IN VARIOUS PHYSICAL SITUATIONS <i>P.D.D.A. Chandralal and M.T.M. Dewasurendra</i>	39
53	FINITE DIFFERENCE METHOD TO SOLVE ONE-DIMENSIONAL SHALLOW WATER EQUATIONS <i>M.M. Zathiha and M.T.M. Dewasurendra</i>	40
59	INCIDENCE COLOURING OF STAR GRAPHS <i>M.M.C Dilshan and A.A.I Perera</i>	41
64	A NOVEL CRYPTOGRAPHIC SCHEME USING THE SUPER MAGIC COVERING OF WHEEL GRAPHS <i>A.M.S.L.K. Athapaththu, R.M.V.V. Bandara and P.G.R.S. Ranasinghe</i>	42

THEME - Mathematics Education

Abstract No.	Title and Authors	Page No.
10	MATHEMATICS TEACHERS' PERCEPTIONS OF FLIPPED CLASSROOM TOWARDS 21ST-CENTURY SKILLS <i>I.S.K. Eriyagama and B.M.S. Bandara</i>	43
26	EXPLORING MATHEMATICAL LANGUAGE OF TEACHERS IN TEACHING CONCEPT OF LIMIT <i>D.R.N.N. Rathnayake and G.N. Jayakody</i>	44
38	EFFECTIVENESS OF MOTIVATIONAL STRATEGIES ON PERFORMANCE IN MATHEMATICS OF GRADE SIX STUDENTS <i>M.P.M. Perera</i>	45
40	ACTIVITY-BASED APPROACH IN TEACHING GEOMETRY FOR LOW ACHIEVING STUDENTS IN GRADE TEN <i>D.M.A. Warushavithana</i>	46
41	OVERCOMING BEHAVIOURAL PROBLEMS OF A GRADE EIGHT STUDENT: AN ACTION RESEARCH <i>H.M.A.B. Herath and W.D. Chandrasena</i>	47
42	FACTORS ASSOCIATING WITH G.C.E. (O/L) EXAMINATION FAILURES IN WESTERN PROVINCE <i>H.N. Perera, A.K. Amarasinghe and W.D. Chandrasena</i>	48
46	RELATIONSHIP BETWEEN PRIMARY TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE AND STUDENTS' ACHIEVEMENT IN MATHEMATICS <i>J.D.T.U.K. Jayasinghe and W.D. Chandrasena</i>	49
48	IMPROVING THE MATHEMATICS RELATED VOCABULARY USING BILINGUAL GLOSSARIES: AN ACTION RESEARCH IN A 1 AB MULTIETHNIC SCHOOL IN CENTRAL PROVINCE <i>M.K. Perera</i>	50
50	THE USE OF GEOGEBRA IN TEACHING AND LEARNING CONCEPTS IN GEOMETRY <i>L.V.P.K. Jayawardana</i>	51
54	TECHNIQUES TO ENHANCE GEOMETRICAL SKILLS OF SLOW LEARNERS: AN ACTION RESEARCH OF GRADE 8 MONOLINGUAL STUDENTS IN A SELECTED SCHOOL IN COLOMBO <i>M.K. Perera</i>	52

THEME - Mathematics Education

Abstract No.	Title and Authors	Page No.
55	FACTORS AFFECTING G.C.E. (ORDINARY LEVEL) FAILURES IN WESTERN PROVINCE, SRI LANKA <i>H.N. Perera, A.K. Amarasinghe and W.D. Chandrasena</i>	53
57	EFFECTIVENESS OF USING IQ QUESTIONS IN ENHANCING PERFORMANCE OF GRADE SIX STUDENTS IN MATHEMATICS: AN ACTION RESEARCH <i>K.R. Wanamalee and W.D. Chandrasena</i>	54
58	ACHIEVEMENT OF COMPETENCY IN MATHEMATICS BY GRADE FIVE STUDENTS: A CASE STUDY IN PATHAHEWAHETA EDUCATION DIVISION, CENTRAL PROVINCE, SRI LANKA <i>D.M.S.D. Dissanayake and W.D. Chandrasena</i>	55
61	EFFECT OF STUDENT ENGAGEMENT AND TEACHERS' EMOTIONAL SUPPORT: A COMPARATIVE STUDY IN PRIMARY AND JUNIOR SECONDARY LEVEL SCHOOLS IN KINNIYA EDUCATION ZONE, EASTERN PROVINCE OF SRI LANKA <i>S.M. Hassan, M.T.M. Dewasurendra and K.M.N.M. Chathuranga</i>	56
62	PEDAGOGY OF MATHEMATICS EDUCATION: A COMPARATIVE STUDY OF EDUCATION SYSTEMS IN SRI LANKA AND FINLAND <i>M.N.M. Siyath, K.M.N.M. Chathuranga and M.T.M. Dewasurendra</i>	57
66	ACTION RESEARCH TO DEVELOP STUDENTS' ABILITY TO USE BASIC MATHEMATICAL SKILLS IN PROBLEM-SOLVING <i>P. Rathnayake</i>	58

Mathematics

GRACEFUL LABELLING OF ROTATED IRREGULAR CYCLIC SNAKE GRAPHS

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Graph labelling is one of the most prominent research areas in graph theory, and the history of graph labelling can be traced back to 1967. There are many graph labelling techniques, such as prime labelling, radio labelling, antimagic labelling, and graceful labelling. Graceful labelling is prominent among these graph labelling techniques since it has a wide range of applications in the real world. In this research, we talk about vertex graceful labelling of graph G . Let f be the vertex graceful labelling function of G which is an injective mapping from $V(G)$ to $[0, |E(G)|]$ such that edge labelling $f_\gamma: E(G) \rightarrow [1, |E(G)|]$ defined by $f_\gamma(uv) = |f(u) - f(v)|$ is also injective. $V(G)$ and $E(G)$ are the set of vertices and the set of edges of G , respectively. In other words, if all the vertices of G can be labelled with the integers from 0 up to the total number of edges of G , $|E(G)|$ in such a way that the induced edge labels get the labelling from 1 up to $|E(G)|$, it is called the graceful labelling of G . Edge labels are computed by taking the absolute difference between the labels of the end vertices of that particular edge. There is a considerable amount of publications related to the graceful labelling of a wide variety of classes of graphs, and there is a famous conjecture related to the graceful labelling of trees, namely, the graceful tree conjecture, which states that all the trees are graceful. This research work focuses on rotated irregular snake graphs, which can be obtained by rotating the main block of a (m, k) C_4 cyclic snake graph by 90° and connecting them where m is the number of inner cycles, and k is the number of blocks. The number of inner cycles in each block can differ from block to block. We obtain formulas for the graceful labelling of rotated irregular cyclic snake graphs and give combinatorial proof.

Keywords: Graceful labeling, Rotated irregular cyclic snake graph, Vertex graceful labeling

TUNING THE PARAMETERS AND SELECTING THE OPERATORS FOR THE GENETIC ALGORITHM TO SOLVE THE VEHICLE ROUTING PROBLEM

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Since the Vehicle Routing Problem (VRP) is proven NP-hard, particularly for large-scale problems, metaheuristic methods are essential to solve and reach near-optimal solutions. The Genetic Algorithm (GA) is considered a suitable metaheuristic method to solve such problems in which some parameters have to be configured at the beginning of the Algorithm and also has some specific operators to be chosen. The values of the parameters have to be adjusted, and the best combination of the GA operators has to be selected to attain the best robustness of the GA. Therefore, this study aims to identify the values of the parameters for the GA and select the best combination of the GA operators using the Taguchi parameters design scheme. In this study, at the first phase, the GA is used to solve VRP, where the factors: population size (levels: 50, 75, 100), number of iterations (levels: 100, 200, 300), termination count (levels: 100, 150, 200), crossover rate (levels: 0.7, 0.8, 0.9) and mutation rate (levels: 0.1, 0.2, 0.3) are considered to design the parameters. In the second phase of this study, another set of factors: selection (methods: Roulette wheel, Rank, Tournament), crossover (methods: Single, Uniform, Order) and mutation (methods: Insertion, Reversion, Swap) are considered to select the best combination for the GA operators. Finally, one-way ANOVA is employed to determine the influential parameters and operators that significantly impact the GA's robustness. It can be concluded that the optimum values for parameters for the GA recommended by the Taguchi experimental design are 50, 100, 100, 0.7 and 0.3 for the population size, number of iterations, termination count, crossover rate and mutation rate, respectively. The outcome of this study concludes that the best combination of the GA operators is Tournament selection, Order crossover and Swap mutation. Moreover, the Termination count, Crossover rate and Mutation rate significantly impact the robustness of the GA in terms of its parameters. In addition, Selection, Crossover and Mutation significantly impact the robustness of the GA concerning its operators to solve the VRP.

Keywords: Genetic Algorithm, Taguchi experimental design, Vehicle routing problem.

STUDY OF NITROGEN DIOXIDE ANNUAL AVERAGE CONCENTRATION IN COLOMBO USING INVERSE DISTANCE WEIGHTED AND LOCAL RADIAL BASIS FUNCTION METHODS

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Urbanization and increment of traffic flow have deteriorated air quality in Sri Lanka's urban areas. Colombo, the commercial capital of Sri Lanka, is a foremost victim of the deteriorated air quality. Since the primary source of air pollution is vehicular emissions, most of the actions recommended were related to controlling vehicular emissions and improving fuel quality. The reduction of Sulfur content and Nitrogen content in diesel and the introduction of unleaded petrol were the actions taken to improve the fuel quality. Much research was conducted, and several actions are being taken to control air pollution in urban areas in Sri Lanka. We studied the annual average Nitrogen Dioxide (NO₂) concentration levels at each sampling location in Colombo during 2013 obtained by Environmental Studies and Services Division, National Building Research Organization (NBRO), Colombo 05. The present study developed an interpolation method to find the annual average NO₂ concentration levels in any location in Colombo using data. We combined the Inverse Distance Weighted technique and Radial Basis Function - Finite Difference method to develop the methodology. The annual average NO₂ concentration levels of sixty selected locations were obtained out of fifteen given NO₂ concentration levels. We could see an error decay when increasing the degree of the augmented polynomials in the radial basis function part.

Keywords: Interpolation method, Inverse Distance Weight Technique, Radial Basis Function-Finite Difference method

MATHEMATICAL MODELLING FOR FINGERO-IMBIBITION DURING CARBONATED WATER FLOODING WITH MAGNETIC FIELD EFFECT

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Fingero-imbibition phenomenon occurs during the enhanced oil recovery process. When a porous medium filled with one phase (oil) comes into touch with another phase (carbonated water) that preferentially wets the medium, the wetting phase (carbonated water) flows into the medium and the native phase (oil) flows out. This occurrence is known as the imbibition phenomenon, and it results from the different wetting capacities of the phases. The imbibition phenomenon, together with the fingering phenomenon, which occurs due to the viscosity difference between the two phases, is known as the fingero-imbibition phenomenon. Carbon-dioxide is a miscible gas with both oil and water, and it tends to reduce the viscosity of oil and enhance its mobility. This allows oil to flow more freely toward the recovery well. Since oil recovery increases with the saturation of injected water, studying this phenomenon is essential. This study is implemented to formulate a new mathematical model to find the carbonated water saturation of the miscible fingero-imbibition phenomenon for an inclined oil layer in a homogeneous porous medium when a magnetic field is present. We obtained a three-term approximate solution using Maple 16 by applying the Method of Directly Defining inverse Mapping (MDDiM), a recent approach to solving nonlinear partial differential equations. We compared the results with and without considering the effect of a magnetic field. Using the obtained results, we can see that the carbonated water saturation of the miscible fingero-imbibition phenomenon increases along with the distance and the inclination angle for a fixed time. Due to the additional magnetic field effect, the saturation of carbonated water is increasing more than when it is without the magnetic field effect. We can conclude that the magnetic field effect plays an essential role in the miscible fingero-imbibition phenomenon, just like the immiscible fingero-imbibition phenomenon.

Keywords: Immiscible fingero-imbibition phenomenon, Inclined oil layer, Magnetic field, Method of Directly Defining the inverse Mapping, Nonlinear partial differential equations

MDDIM SOLUTIONS IN SOLVING NONLINEAR FUZZY HEAT-LIKE EQUATIONS

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Phenomena of nature or physical systems can be described using Partial Differential Equations (PDEs), such as wave equations, heat equations, and Poisson's equation. As a result, investigations of PDEs have become one of the critical areas of modern mathematical analysis, attracting much attention. The Method of Directly Defining the inverse Mapping (MDDiM) is a recent approach to solving nonlinear differential equations that was introduced by S. Liao in 2016 to solve nonlinear single Ordinary Differential Equations (ODEs), and it was obtained by directly defining the inverse linear map to the Optimal Homotopy Analysis Method (OHAM). In the frame of MDDiM, we have the great privilege of choosing the inverse linear operator without choosing the auxiliary linear operator. Dewasurendra et al. extended this so-called method to solve a system of coupled nonlinear ODEs, and recently we further developed it to solve single and coupled nonlinear Partial Differential Equations (PDEs). In this paper, we extend MDDiM to solve a one-dimensional Initial Value Problem (IVP) describing a heat-like equation, a single nonlinear Fuzzy Partial Differential Equation (FPDE). Zadeh introduced fuzzy sets, and hundreds of examples have shown that the nature of the uncertainty in the behaviour of certain system processes is fuzzy rather than stochastic. Many authors have recently expressed an interest in researching the theoretical framework of fuzzy IVPs. Using Maple 16, we generated three and four-term approximations for the fuzzy heat-like equation, and we compared the outcomes with the exact solution and the order. All the solutions have minimal squared residual errors and are sufficiently accurate.

Keywords: Heat-like equations, Initial value problems, Method of Directly Defining the inverse Mapping, Nonlinear fuzzy PDEs

**A MATHEMATICAL APPROACH TO FINDING THE OPTIMAL WORKING
ROLE ALLOCATION WHICH MINIMIZES THE TOTAL WORKING TIME**

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Workforce time management is an essential task for the management decision-makers of any industry. In most industries, they are trying to prepare the optimal schedule to minimize the working time and increase the production efficiency of the working floor. Many researchers have tried to solve the Assignment Problem (AP) efficiently and accurately. Harold Kuhn developed an algorithm in 1955 known as the Hungarian Algorithm. AP has numerous industrial applications to control and optimize scheduling and job allocation activities. In this investigation, our focus was to find the optimal working role allocation to each machine-line on the working floor of the *ABC* garment company, which has a medium-scale production capacity in Western Province, Sri Lanka. This allocation is being made manually by the work superintendent for each month. After scrutinizing the assignment schedule, a few drawbacks were encountered, such as poor allocation of working-roles. Also, the current assignment schedule does not fulfil certain requirements that *ABC*'s Management expects. This motivated us to conduct the investigation to overcome these prevailing drawbacks. The analysis was conducted on data received in 2022 from the work-superintendent of *ABC* garment company. An assignment model was developed to optimize the allocated workers' total working time, which helps to minimize the total daily cost which occurs due to the workers' salaries. The solution was compared with the existing assignment schedule, revealing that *ABC* can save up to 120,960 LKR annually if the proposed solution is implemented. Moreover, *ABC* does not have to make any additional effort to implement the suggested method but reduce a significant amount of cost by minimizing the total working hours of employees. Therefore, the outcome obtained by solving the model is very rational and encouraging.

Keywords: Assignment model, Minimization problem, Optimum assignment schedule

**FORECASTING AND EXAMINATION OF THE DYNAMIC BEHAVIOUR OF THE
GLOBAL FINANCIAL DEVELOPMENT INDEX**

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Economists, researchers, and financial analysts frequently use vector autoregressive (VAR) models to study time series variables' dynamic behaviour. The impulse response function (IRF) plays a major role in the vector autoregressive models in examining the response of the model's variables to a shock. Aside from that, VAR models offer unmatched forecasting and prediction capabilities for financial and economic data. Annual financial indices from 1980 to 2020, published by the International Monetary Fund to assess the development of financial institutions and markets in terms of depth, accessibility, and efficiency, were considered endogenous variables. The dynamic behaviour of the data was examined through IRF and Granger causality analyses. The financial development index (FDI) was forecasted over the next ten years with 95% confidence using the VAR model. The VAR (9) model with the endogenous variables of financial development and financial institutions' efficiency exhibited the lowest mean absolute scaled error of 0.279. Granger causality results agreed with the selection of endogenous variables in the optimal VAR (9) model. The impulse of one standard deviation in the financial institution efficiency index resulted in a spike in the FDI after 2 years. Moreover, fluctuations observed within the FDI to the impulse of the financial institution efficiency index were closely associated. VAR, Granger causality, and IRF results emphasized the significance of financial institutions' efficiency in global financial development.

Keywords: Financial Institution Efficiency Index, Granger Causality, Impulse Response Function, Vector Autoregressive Model

COMPUTING h -FUNCTION OF A DOUBLY CONNECTED REGION FORMED BY DELETING A SLIT FROM THE UNIT DISC

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Consider a two-dimensional region Ω and a fixed point z_0 in Ω . The harmonic-measure distribution function, or h -function, represents the probability that a Brownian traveller released from z_0 first leaves the region Ω somewhere within distance r of z_0 . The h -function $h(r)$ is a non-decreasing, right-continuous function and lies between the values 0 and 1. The focus of this study is the computation of the h -function of a doubly connected bounded region Ω formed by deleting an interval from the unit disc. In the computation of $h(r)$ for this region, we use the radial-slit Mapping and the Cayley-type mappings in terms of the prime function. To obtain the formula for $h(r)$ of this region Ω with a basepoint z_0 , we first identify a concentric annulus as the preimage region D_ζ for the region Ω , using appropriate conformal transformations, including a radial slit map. Next, we use a suitable Cayley-type map that transforms the region D_ζ onto the lower halfplane with a deleted slit which emanates from the origin and lies in the lower halfplane. Finally, for each range of r , we formulate appropriate potential function $W(\zeta)$ whose imaginary part $Im[W(\zeta)]$ is harmonic on D_ζ , and solves our Dirichlet problem in D_ζ . Also, the h -function $h(r)$ is equal to $Im[W(\zeta)]$ at the preimage ζ_0 of the basepoint z_0 .

Keywords: h -function, Cayley-type maps, Prime function, Radial-slit mapping

A FRACTIONAL ORDER MATHEMATICAL MODEL FOR THE SPREAD OF COVID-19 WITH QUARANTINE AND ISOLATION CONTROL STRATEGIES

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Infectious diseases are disorders caused by organisms such as bacteria, viruses, fungi or parasites. These infectious diseases can cause an epidemic or pandemic. COVID-19 is an infectious disease caused by the SARS-Cov-2 virus, which spread up to a world pandemic. The covid-19 pandemic is more than a health crisis affecting societies and economies. It is most likely to increase poverty and inequalities globally; hence, every country needs to act immediately to prepare, respond and recover. For that, understanding the transmission dynamics of the disease through mathematical modelling is useful. According to the literature, it is accepted that the fractional order models are more accurate and reliable than the classical integer order models, and the fractional order model cooperatively better fits the actual data. Therefore, we formulate a system of fractional order mathematical model including the subclasses susceptible (S), exposed (E), quarantined (Q), infected (I), isolated (J) and recovered (R) named SEQIJR model to understand the transmission of the covid-19 considering control strategies quarantining and isolation. We assume that the total population remains constant, and the model can be applied after implementing the quarantining and isolation strategies. Further, we perform a stability analysis and derive the basic reproduction number of the model. Furthermore, numerical solutions of the proposed model are obtained using the FDE-12 MATLAB open software, which implements the Adam-Bashforth-Moulton method. Finally, the impact of the fractional order for the population sizes of some key components of the model is discussed. In this study, we did not have actual data on coronavirus, but in future research, we expect to do the parameter estimation to find the most suitable fractional-order value and model parameters in which the actual data fits with simulated data.

Keywords: Adam-Bashforth-Moulton method, Basic reproduction number, Fractional derivatives, Stability analysis

ODD PRIME LABELLING OF DOUBLE SNAKE GRAPH

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Graph theory is a branch of mathematics that deals with relationships between objects, called vertices, and connecting lines, called edges. Since the 1960s, graph labelling has been a leading research area in graph theory. Graph labelling is a process of assigning labels or values to the vertices or edges of a graph. These labels can represent information about the graph or help solve particular problems related to the graph. There are several types of graph labelling, including vertex labelling, edge labelling, and face labelling. Also, there are various graph labelling techniques in graph theory, such as prime labelling, radio labelling, graceful labelling, lucky labelling, and antimagic labelling. In this research, we use one of the variations of prime labelling called odd prime labelling of double snake graphs. Odd prime labelling is a type of vertex labelling in graph theory where each vertex in the graph is assigned a set of odd integers $(1, 3, \dots, 2n - 1)$ as a label so that any two adjacent vertices are relatively prime. A double-snake graph introduced by this paper is a mathematical graph in which the vertices are arranged in a "snake-like" pattern. As a linear pattern, the vertices of the first ring connect with those of the next ring. Labelled graphs are helpful in mathematical models for a wide range of applications, such as coding theory, circuit theory, computer networks, and cryptography. This paper deals with an undirected, simple, and connected double-snake graph $B_{n,m}$ where n denotes the number of cycles in the graph for any $n \geq 1$ and where m denotes the number of vertices in the one upper or lower cycle, which is the number of vertices in the upper and lower cycle that are equal and for any $m \geq 1$, according to the varying of n and m , the double-snake graph also varies. Our paper proves that the double snake graph can be labelled using odd prime labelling for any n and m , and introduces a general theorem. In this research work, we obtain a generalized method for obtaining odd prime labelling for a double snake graph and give combinatorial proof for it.

Keywords: Double snake graph, Odd prime labelling, Snake graph

AN ALGORITHM FOR RAINBOW COLOURING OF THE HIGHER ORDER EXTENSION OF THE SANDAT GRAPH

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Graph labelling is one of the most intriguing areas in graph theory, and graph colouring is a special case of graph labelling with assigning colours to edges or vertices of the graph. Edge colouring assigns colours to each edge in a graph so that no two adjacent edges have the same colour with a given optimal number of colours. Edge colouring is proper edge colouring if adjacent edges are coloured with distinct colours. A rainbow path is a path in an edge-coloured graph with no two edges having the same colour. If every pair of vertices in a graph are connected by at least one rainbow path, then that graph is a rainbow-connected graph. The minimum number of colours used in a rainbow-connected graph is the rainbow connection number ($rc(G)$) of that graph. Since graph colouring applications appear in diverse fields, such as Computer science, it is essential to introduce new graph classes. A new graph called (Sandat Graph ($St(n)$)) was introduced by Fredlina et al. in 2020. In the present study, we have introduced a higher order extension of the Sandat graph as an extended version of the Sandat graph defined by $SSt_m(n)$ having n number of petals using the symmetrical subdivisions. Each petal has $2(2 + m)$; $m \in \{1, 2, 3, \dots\}$ number of vertices. This graph has the vertex set $V(SSt_m(n))$ and the edge set $E(SSt_m(n))$ denoted by, $V(SSt_m(n)) = \{r, s_{ij}^h, t_i; 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq h \leq m + 1\}$ and $E(SSt_m(n)) = \{rt_i, rs_{ij}^h, t_i s_{ij}^1, s_{ij}^k s_{ij}^{k+1}; 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq h \leq m + 1, 1 \leq k \leq m\}$. An algorithm was proposed for rainbow colouring using only three colours for the above graph, and it remains to prove the rainbow connection number of the above graph as three.

Keywords: Edge colouring, Rainbow colouring, Rainbow connection number, Sandat graph

SOME TOPOLOGICAL INDICES OF FIRECRACKER GRAPHS

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Chemical graph theory has significantly contributed to understanding the properties of chemical compounds in the modern era of chemical science. One of the key areas of chemical graph theory is to calculate the topological indices of molecule graphs. Topological indices play a crucial role in predicting various physicochemical properties of molecules. Because various physicochemical properties of molecules can be predicted using the topological index value. This study investigates the structural characteristics of firecracker graphs. A firecracker graph Denoted by $F_{n,k}$ is a tree consisting of n path and k bunch of stars, where each vertex on the path graph is joined by the root of exactly one-star graph, $n \geq 2, k \geq 4$. Firecracker graph plays a vital role in Network theory. A firecracker graph can be used to design the networks whose efficiency is maximum, and the optimum path is available. Finding chromatic numbers, applying graph labelling techniques, applying colouring techniques, and finding some degree-based topological indices for the line graph of the firecracker graph can be seen in the literature. E. Deutsch and S. Klavžar introduced M-Polynomial, considered the most general polynomial to produce many degree-based topological indices. We derive formulas for a range of topological indices, including the First, Second, and Third Zagreb Indices, Hyper-Zagreb Index, Randić Index, Atom-Bond Connectivity Index, Geometric-Arithmetic Index, Sum-Connectivity Index, Inverse Sum Index, for the firecracker graph $F_{n,k}$. The findings provide valuable insights into the structural characteristics and properties of firecracker graphs, further advancing the field of graph theory and topology in chemical science.

Keywords: Firecracker graph, M-polynomial, Topological indices

**THE ANT COLONY OPTIMIZATION ALGORITHM TO SOLVE THE
TRAPEZOIDAL TRANSPORTATION PROBLEMS**

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The transportation problem is the most appealing and well-known problem in operations research. Different techniques have been developed in the past to solve the transportation problem. Nevertheless, in real life, parameters may be vague or uncertain, with some limits. Project scheduling, assignment problems, and network traffic problems are examples of real-world problems that optimization techniques can solve. A novel model of the transportation problem known as "fuzzy transportation problems" (FTP) was created as the incorporation of an uncertain environment. With the assistance of the above concept, a fuzzy transportation problem is created, which can be defined as having unspecific quantities concerning multi-objective reasoning. Various studies were conducted in the literature to transform the fuzzy transportation problem into a crisp problem with different mathematical models. This paper mainly focuses on resolving the fuzzy trapezoidal transportation problem using a novel form of the Ant Colony Optimization Algorithm (ACOA). A novel method is proposed for determining the type of FTP in which the transportation expenses and values of supplies and demands are represented by trapezoidal fuzzy numbers to address the shortcomings of some existing methods. The modified ACOA is used to solve transportation problems derived from the FTP. By reforming ACOA and incorporating the Transition Rule and Pheromone Update Rule, degree of satisfaction with the optimal solution has increased in this novel method. Comparing the algorithmic method proposed by this method to the well-known meta-heuristic algorithms in the literature, this study is less complex. The case study is used to demonstrate the proposed approach.

Keywords: Ant Colony Optimization Algorithm, Arithmetic Mean, Fuzzy Transportation Problem, Initial Feasible Solution, Trapezoidal Fuzzy Number

RADIO ARITHMETIC MEAN LABELLING OF PENDANT GRAPHS

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A graph G is a pair of sets $(V(G), E(G))$, where $V(G)$ and $E(G)$ are the set of vertices and the set of edges in G , connecting the pair of vertices, respectively. Graph labelling is one of the most popular research areas in graph theory. There is a vast amount of literature available on graph labelling. Radio labelling is one of the practical methods among the other graph labelling methods. This labelling method is used for channel assignment problem where we must both avoid signal interference between radio stations and minimize the range of frequencies used. Hale's radio frequency classification problems were implemented in 1980. Motivated by channel assignment of FM radio stations, Chartand defined the concept of radio labelling of graph G in 2001. Ponraj introduced the notion of radio mean labelling of G , and they found the radio mean number of graphs and subdivided graphs. Telecommunications, biology, and physics are some areas in which radio labelling can be applied. A radio labelling f of G is an assignment of positive integers to the vertices of G satisfying, $|f(u) - f(v)| \geq \text{diam}(G) + 1 - d(u, v)$, where $u, v \in V(G)$, $d(u, v)$ is the distance between any two vertices in the graph and the maximum distance between any pair of vertices is called the diameter of G denoted by $\text{diam}(G)$. The above result is generalized by including the arithmetic mean of labeling f instead of their difference, such that $\lceil \frac{f(u)+f(v)}{2} \rceil \geq \text{diam}(G) + 1 - d(u, v)$. The radio mean number of f , $\text{rmn}(f)$, is the maximum number assigned to any vertex of G . The radio mean number of G , $\text{rmn}(G)$ is the minimum value of $\text{rmn}(f)$ taken over all radio mean labellings f of G . An alternative proof for radio mean number of pendant graph, $C_n \odot K_1$ where $n \geq 3$ is presented in this research work. In our proof, we have grouped the pendant graphs into 4 categories, pendant graphs with even cycle and even diameter or odd diameter, and pendant graphs with odd cycle and even diameter or odd diameter.

Keywords: Pendant Graph, Radio Arithmetic Mean Labelling, Radio Mean Number

**A LOCAL SEARCH-BASED HYBRID GENETIC ALGORITHM FOR THE
VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICKUP AND
DELIVERY**

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The Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD) is a crucial problem in transportation. It is a special variant of the traditional Vehicle Routing Problem (VRP). In VRPSPD, all commodities must either begin or end at the depot, and pickup and delivery activities of customers are performed simultaneously. The VRPSPD is more complex than the traditional VRP and belongs to the NP-hard category. Therefore, metaheuristic approaches are generally used to solve VRPSPD in the literature. This study developed a hybrid metaheuristic algorithm for solving VRPSPD by integrating the Local Search (LS) procedure into the Genetic Algorithm (GA). The proposed hybrid metaheuristic has three versions based on how the LS technique is integrated into the GA, and they are denoted as PHGA-P1, PHGA-P2, and PHGA-P3. In the PHGA-P1 algorithm, the LS technique is only used to improve the initial population of the GA. However, the LS technique is used twice in the PHGA-P2 algorithm to improve the initial population and to expand the search space after the crossover operation. In the PHGA-P3 algorithm, the LS technique is used at the end of the mutation operation in addition to the integrations of the LS technique in the PHGA-P2 algorithm. The performance of the hybrid algorithms was compared with the general GA and the modified GA introduced by Rahimi in 2016 using a set of standard benchmark problem instances. In comparison, the total travelling distance of solutions and the CPU time required to reach the solution was considered. According to the comparison results, the proposed three hybrid algorithms are significantly better than the general and modified GA in terms of total travelling distance. However, the PHGA-P2 and PHGA-P3 consume more CPU time than other algorithms. Therefore, the proposed hybrid Algorithm PHGA-P1 can be recommended to solve the VRPSPD.

Keywords: Hybrid Genetic Algorithm, Local Search, Simultaneous Pickup and Delivery, Vehicle Routing Problem

IMMUNITY WANING DYNAMICS OF COVID-19 THROUGH A MATHEMATICAL MODEL

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Since early 2021, people have been receiving vaccines in first, second, and third doses to build up their Immunity against COVID-19. However, the Immunity of vaccinated individuals gradually diminishes over time. That means the innovative vaccines for COVID-19 are imperfect. Therefore, presently, the COVID-19 virus is resurging due to the waning Immunity of the vaccinated people. In this paper, we focused on population behaviour and dynamics of disease spreading with Immunity waning rates. For that, a new mathematical model is proposed, consisting of seven compartments; susceptible (S), immunised by only one dose of vaccine ($V1$), immunised by only two doses of vaccine ($V2$), immunised by all three doses of vaccine ($V3$), infected (I), hospital isolated (H), and recovered (R). Based on the matrix operator method, the disease-free reproduction number, R_{df} , of the model is derived. We analysed the local stability and observed that disease-free equilibrium is stable if $R_{df} < 1$ and unstable if $R_{df} > 1$, and endemic equilibrium is unstable if $R_{df} < 1$ and stable if $R_{df} > 1$. We estimated the effectiveness of the model parameters on the spread of the disease by analyzing the sensitivity of R_{df} . Based on those results, we observed that the transmission rate from the susceptible class to the infected class (β) is the most influenced parameter, and the next influenced parameter is the recovered rate of the infected population (γ_1). This research reveals that as the Immunity waning rates increase, the size of the infected population increases, and the size of the immunised populations decreases. Also, if a higher percentage of the population receives three doses of the vaccine, the increment of the infected population becomes low.

Keywords: Disease-free reproduction number, Immunity Waning, Sensitivity analysis, Vaccine doses

AN INTEGER LINEAR PROGRAMMING MODEL TO OPTIMIZE NURSE SCHEDULING: A CASE STUDY

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Nurse Scheduling (NS) is a well-known optimization problem in Operations Research. It can be defined as assigning nurses for well-defined shifts for a specific period satisfying a set of conditions related to rules and regulations stipulated by the hospital management and nurses' preferences. Developing a highly efficient and effective NS is a difficult task. The deployed nurse workforce and the hospital's policies highly affect nurses' working conditions and the quality of their service. Scheduling in healthcare is often done manually, which is time-consuming and does not always give the optimum schedule. This study proposes an automated scheduling method for a planning period based on a 0-1 integer linear programming model to overcome these inefficiencies. It satisfies the set of constraints associated with the hospital and nursing staff. Several studies have attempted to minimize the number of shifts during a day, the sum of penalty costs associated with each nurse, the number of nurses in deficit, and the differences between the number of hours that each nurse works during a month and the number of hours the nurse has to work. The proposed mathematical model generates an optimal nurse schedule for a predetermined period in an average of 3 seconds of computational time. The nursing staff considered in this case study falls into several categories, and also, a day has several shifts. The decision variables in this model determine whether each type of nurse in the staff is assigned to a shift or not. The objective function of the model attempts to reduce the hospital's operational cost, the cost incurred with the overtime payments, and the regular payment of the nurses, subject to constraints that are associated with the hospital rules and essentials. The proposed model incorporates the preferences of the working shifts of the nurses. Also, this model can be easily extended to the NS of any hospital with similar rules and regulations governing nursing staff. This method is practically useful to the hospital management to prepare a monthly nurse schedule and also to make necessary changes in contingency situations. Since the overtime payments are higher than that of the regular payments, it is desirable to reduce overtime. It can be observed that over time has been reduced considerably in the proposed schedule.

Keywords: Nurse Scheduling Problem, Integer linear programming

**NEW EXPLICIT FORMULAS FOR HIGHER-ORDER APPROXIMATIONS OF
FIRST AND SECOND DERIVATIVES WITH APPLICATIONS**

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Numerous applications of the first and second derivatives abound in many areas, including science, engineering, economics, mathematics, medicine, and optimization, and have demanded higher-order accurate computational techniques to solve derivative-based problems such as first and second-order differential equations. The finite difference method is a computational technique widely used to discretize derivatives, in contrast to other methods such as finite element and finite volume methods. Linear combinations of the Taylor series expansions of a function at various grid points about the point of a desired derivative are often devised to derive finite difference weights. For higher-order approximations, this process includes heavy hand computations and solving large linear systems, making computational procedures cumbersome. This study proposes two higher-order weighted average finite difference approximations for the first and second derivatives. To attain this, using the standard second-order central difference approximations for the first and second derivatives, two higher-order weighted average finite difference approximation operators are derived for these derivatives. The operators, including ap number of central differences, approximate the derivatives with an accuracy of order $2p$. Further, two explicit formulas are constructed for the weights of these operators. The approximations are applied to the second-order boundary value problem. Some numerical tests are presented to demonstrate the effects of the approximations. These explicit formulations are desirable to automate the approximations for the first and second derivatives.

Keywords: Explicit formulas, Finite difference method, First and second derivatives, Second order boundary value problem

CHROMATIC POLYNOMIAL OF A SPECIAL TYPES OF LADDER GRAPH

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The chromatic polynomial of a graph G , denoted by $P_G(k)$, is the function that takes in a non-negative integer k as input and returns the number of ways to colour the vertices of G with k distinct colours so that no two adjacent vertices are assigned the same colour. The initial definition of the chromatic polynomial came from George David Birkhoff, which was used to study four colour problems in 1912. Naturally, the chromatic polynomial is often used in graph colouring problems, widely applicable in resource allocation, scheduling, and pattern matching. In our work, we focus on the Millipede graph ($M_{\mathbb{Z}}$) with $4n$ vertices and $5n - 2$ edges. The Millipede graph is a special type of ladder graph. The most popular theorem to obtain chromatic polynomials $P_G(k)$ of a graph, G , is the deletion-contraction theorem. In this study, we split the millipede graph into two sub-graphs along an edge to calculate chromatic polynomials. There are some general formulas for the chromatic polynomial of some special type of ladder graph. However, most of them have not been proven. In our work, the general formula for the chromatic polynomial of the Millipede graph is obtained, and the correctness of the general formula is established by using mathematical induction on the number of vertices. The general formula for the chromatic polynomials of the millipede graph is determined as, $P(M_{\mathbb{Z}}, k) = k(k - 1)^{2n+1}(k^2 - 3k + 3)^{n-1}$, where $4n$ is the number of vertices.

Keywords: Chromatic polynomial, Ladder graph, Millipede graph

A NOVEL SECOND-ORDER FINITE DIFFERENCE APPROXIMATION FOR FISHER EQUATION

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Fisher Equation (FE) is a nonlinear partial differential equation that models many physical systems involving the effects of both linear diffusions and nonlinear reactions. The FE arises in numerous applications, including brain tumour dynamics, population dynamics, and chemical reactions. In the literature, computational techniques have been preferred to attain a solution to the FE due to the lack of closed-form solutions of analytical methods. Among several computational techniques, the finite difference approximation method has widely been used to obtain discrete solutions of the FE. To achieve higher accuracy, some second and fourth-order accurate finite difference approximations for the FE have been derived in the literature by combining them with Crank-Nicholson (CN) Technique. These methods discretize the nonlinear part of the FE into its corresponding linear form using the lagging technique. The approximation order of the lagging linearization is one for the time discretization, and consequently, the overall approximation order of the time discretization of the foregoing CN schemes brings down to one. Therefore, the CN schemes do not give the expected accuracy. This study aims to develop a new second-order finite difference scheme for the FE, a new linearization technique. By using the concepts of arithmetic and geometric means, the new linearization technique is derived to discretize the nonlinear part of the FE at the middle point of two consecutive time steps with order 2 accuracy. Using this linearization technique, a new CN scheme is obtained. The new scheme is second-order accurate in time and space. Numerical tests are carried out to demonstrate the effectiveness of the proposed scheme. The results show that the proposed scheme for the FE is more accurate than the existing lagging technique-based second and fourth-order CN schemes. Furthermore, the stability of the CN scheme is numerically confirmed using the spectral radius of the iteration matrix of the new CN scheme.

Keywords: Accuracy, Arithmetic mean, Crank-Nicolson method, Geometric mean, Lagging technique

SOME PROPERTIES OF FUZZY METRIC SPACES

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The notion of fuzzy metric spaces has many advantages in mathematical analysis since many notations and results from classical metric space theory and topological space theory can be extended and generalized to the setting-up of fuzzy metric spaces. In this research paper, we used fuzzy metric spaces, triangular norm condition, openness and fuzzy topology to establish some properties. In this direction, we proved that when $B(x, r_1, t)$ and $B(x, r_2, t)$ are two open balls with same center $x \in X$ and $t > 0$, with radii satisfying $0 < r_1 < 1$ and $0 < r_2 < 1$ respectively, then either $B(x, r_1, t) \subseteq B(x, r_2, t)$ or $B(x, r_2, t) \subseteq B(x, r_1, t)$; Every open ball in a fuzzy metric space $(X, M, *)$ is an open set; Every fuzzy metric space is a Hausdorff space. Moreover, when (X, d) is a metric space, then $M_d(x, y, t) = \frac{t}{t+d(x,y)}$ is the corresponding standard fuzzymetric on X . Then the topology τ_d induced by the metric d and topology τ_{M_d} induced by the fuzzy metric M_d are the same. Next, we provide some examples on the underlying set $X = N$; let $M(x, y, t) = \{\frac{x}{y}, x \leq y \frac{y}{x}, y \leq x, \text{ where } t > 0$. Then $(N, M, *)$ is a fuzzy metric space under product triangular norm ($a * b = ab$) but $(N, M, *)$ is not a fuzzy metric space under minimum triangular norm ($a * b = \{a, b\}$). Note that $M_d(x, y, t) = \frac{t}{t+d(x,y)}$ is not a fuzzy metric space under minimum triangular norm condition ($a * b = \{a, b\}$) for any induced metric structure (X, d) . **Keywords:** Fuzzy metric space, Hausdorff space, Topological space, Triangular norm

A DATA-DRIVEN APPROACH FOR SIGNATURE VERIFICATION

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The signature of an individual is a unique key that may change due to some factors like age, mood, and environment. A Signature Verification System (SVS) is a solution to find whether the questioned document and the known document are from the same person or not. Here we have worked with an offline signature verification based on geometric features. This paper proposes a novel approach for SVS using Reduced Order Modeling (ROM) based on Proper Orthogonal Decomposition (POD). POD is a mathematical approach that converts the high dimensional data into a lower dimensional model, which extracts essential features that represent the main characteristic features of the original data set. To create the ROM and test the model performances, we consider 30 different actual signatures as our training data set, including five genuine signatures from each six different signature types. The main objective of this study is to test the performance of the ROM by reconstructing an input signature and verifying the signatures in the test data set as genuine or forgeries. The ROM's required basis functions are obtained using the proper orthogonal decomposition. The eigenvalue spectrum is used to obtain the required number of basis functions. When we select the number of dominant eigenvalues as 30, the reduced order model can successfully reconstruct the signatures. The signature's quality is tested using the structural similarity index measure. The reduced order model yielded a similarity index value of 0.6494. As this value is greater than 0.5, we can conclude that ROM produces quality reconstructed signatures. To do the verification process, we have trained 180 signatures which included 18 genuine signatures from each 10 different signature types. The error in the training data set was found using the singular values. An index and confidence interval relevant to the training data was found by replacing 40 genuine signatures randomly. Based on the index and the confidence interval, 60 signatures were tested as genuine and forgeries. True positive and false positive rates are used to measure the model accuracy at the significant levels of 90%, 95%, 99%, and 99.9%. At a 99.9% confidence interval, the proposed technique gives the results with 64% accuracy.

Keywords: Proper orthogonal decomposition, Reduced order model, Signature verification

PORTFOLIO REBALANCING IN THE PRESENCE OF TRANSACTION COSTS

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The Modern Portfolio Theory (MPT) reduces portfolio risk by choosing and balancing assets based on statistical methods that measure the level of diversity by computing expected returns and standard deviations of individual securities to assess their risk. Rebalancing is the process of restoring the asset allocation values of a portfolio to the levels specified by an investment strategy. Rebalanced portfolios will help investors to engage in financial market trading to reach their objectives in the most practical manner. This study focuses on finding an efficient portfolio and rebalancing it with fixed transaction costs. In this study, yahoo finance was used to access historical close prices for 10 randomly chosen stocks from the Financial Times Stock Exchange 100 Index (FTSE100) over a five-year period. MATLAB software was used to build a portfolio that includes a risk-free rate, asset return moments, and an estimate of the mean covariance of each return. The best-diversified weights among the three efficient portfolios were found in the parametric efficient portfolio, which was chosen for rebalancing. Before rebalancing, the parametric efficient portfolio's return and risk were 0.0104 and 0.0360, respectively. To rebalance the portfolio, considering transaction costs, we employed a quadratic programming model to determine the optimal weights and traded quantities for each investment. Through analyzing the impact of adjustments in buying/selling amounts and transaction costs, we observed a consistent decrease in the portfolio's risk and return. These findings provide investors with a valuable framework for constructing and managing efficient portfolios, enabling informed decision-making within financial markets. To ascertain whether variation in risk and return is the same, select to apply this model to data from a different stock market in a later study.

Keywords: Modern Portfolio Theory, Parametric efficient portfolio, Rebalance, Transaction costs

THE EXTENDED AUXILIARY EQUATION METHOD FOR THE NONLINEAR SCHRÖDINGER EQUATION WITH VARIABLE -COEFFICIENT

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In this paper, we investigate the exact solutions of the nonlinear Schrödinger equations with variable-coefficient. The nonlinear Schrödinger equations describe wave propagation in optical fibers with nonlinear impacts. Finding the exact solutions of nonlinear equations plays an important role in nonlinear science, especially in engineering and mathematical physics. In recent years, many methods have been used to find the exact solutions of nonlinear equations. One of the most effective techniques, namely, the extended auxiliary equation method is utilized to obtain the exact solutions of the considered equations. As a result, solitary wave solutions, trigonometric function solutions, rational function solutions, and Jacobi elliptic doubly periodic wave solutions are obtained. Further, some of the obtained solutions are presented by 3D and 2D graphs to demonstrate the behavior of solutions. The results show that the extended auxiliary equation method is reliable and effective in finding various exact solutions of nonlinear evolution equations with variable coefficients in mathematical physics. The resulting solutions are novel, intriguing, and potentially helpful in understanding the phenomena in which waves are governed by such equations. In future, we aim to extend the proposed method for tackling some other nonlinear models.

Keywords: Extended auxiliary equation method; Solitary wave solutions; Nonlinear Schrödinger equations.

**A NEW FOURTH-ORDER CRANK-NICOLSON DIFFERENCE APPROXIMATION
FOR THE DIFFUSION EQUATION**

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The one-dimensional diffusion equation (DE) models the distribution of heat in a uniform rod over time. The DE is a linear parabolic partial differential equation with numerous applications in physics, chemistry, finance, geology and other fields. Analytical solutions of the DE are often cumbersome, and thus computational or numerical methods are more commonly used. The finite difference method is a computational technique which is widely being used to solve the DE. There are three basic finite difference approximations; explicit, implicit, and Crank-Nicolson methods, depending on how the time derivative of the DE is discretized. Since the Crank-Nicolson method is unconditionally stable and second-order accurate in both time and space, it is highly accurate than the other methods. This study aims to develop higher order (> 2) approximation for the DE using Crank-Nicolson type methods to approximate solutions to DEs. A new fourth-order Crank-Nicolson difference approximation is designed for the DE. To attain this, the spatial second derivative (SD2) of the DE is formed to have a convex combination of two second-order accurate central difference approximations of the SD2. The convex coefficients are determined to have a fourth-order approximation for SD2. Using the Crank-Nicolson technique and the preceding fourth-order approximation, a new fourth-order Crank-Nicolson approximation with two shift parameters is derived. The new approximation is spatially fourth-order accurate and temporally second-order accurate. Each pair of unequal integer shifts yields a unique fourth-order Crank-Nicolson scheme. The numerical results of the fourth-order Crank-Nicolson scheme do not provide better solutions when shift values are (1,2) or greater. Theoretically, the proposed scheme has a higher order than the standard Crank-Nicolson scheme, but numerically, the standard scheme provides solutions with higher accuracy than the proposed scheme. The reason could be that the iteration matrix of the new Crank-Nicolson scheme is singular or that the Crank-Nicolson scheme is conditionally stable. The stability analysis of the fourth-order scheme is the subject of further study. Therefore, it can be concluded that the standard Crank-Nicolson scheme is a better approximation method than the proposed fourth-order Crank-Nicolson scheme.

Keywords: Crank-Nicolson scheme, Diffusion equation, Fourth-order approximation, Shift parameters

A TWO-SEX AGE-STRUCTURED POPULATION DYNAMICS MODEL FOR THALASSEMIA TRANSMISSION

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Thalassemia is an inherited blood disorder in which the rate of hemoglobin production is wholly or partially suppressed due to the reduced rate of synthesis of alpha-like or beta-like globin chains, the two chains of adult human hemoglobin, leading to various clinical manifestations. It has burdened the healthcare systems of many countries in the Mediterranean basin and tropical or subtropical regions of Asia and Africa. Due to the migration of people from these regions, thalassemia populations have also become a public health concern in other countries. As a result of this, thalassemia will be an essential factor in community health concerns in the next century. Therefore, attention must be paid to the correlation between the evolution of the population and the prevalence of thalassemia. Structured models are required for populations of multicellular organisms and diseases in which age and sex play an essential role in the transmission dynamics. The main objective of this research is to develop a two-sex age-structured continuous-type population dynamics model that describes the genotype composition of the population resulting from the thalassemia trait. In modelling the dynamics, the whole population is classified into several components based on sex, marital status, age, and the genotype encountered in studies of thalassemia. The model is formulated based on the Fredrickson-Hoppensteadt model, consisting of semi-linear partial differential equations with nonlocal boundary conditions. The modelling process here is entirely from standard infectious diseases modelling techniques, as transmission from parents to offspring depends on the genotype of the parents. The existence and uniqueness of a continuous solution to the proposed model are established by applying the integral equation approach under the biologically relevant conditions of the model's parameters. The presented model has the potential to be helpful in both mathematical and biological aspects.

Keywords: Age structured, Population dynamics, Thalassemia

**ARTIFICIAL BEE COLONY ALGORITHM TO SOLVE LINEAR FRACTIONAL
TRANSPORTATION PROBLEM WITH VARYING DEMAND AND SUPPLY**

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The Linear Fractional Transportation Problem (LFTP) plays a vital role in logistics and supply management to reduce cost and improve service. The objective of the LFTP is to minimize the total costs-to-total profit ratio. It is formulated as a linear fractional programming problem. Studies were conducted to develop approaches to solve LFTP by various researchers. However, in real-world scenarios, demand and supply quantities are not deterministic. They are stochastic and vary within a range due to global economic conditions such as price, availability, and quality of a product. Due to demand and supply variations, the minimum total cost to total profit ratio also varies in a specific range. A study was conducted to develop an approach to solve LFTP with varying demand and supply (LFTPVDS) quantities by Joshi & Gupta (2011). In that study, the lower and upper bounds of the minimum total cost to total profit ratios were investigated. This paper also introduces an efficient approach using the Artificial Bee Colony (ABC) algorithm to find out the lower and upper minimal objective function values of LFTPVDS. ABC algorithm is a swarm intelligence algorithm. It is used to solve more complex problems due to its specific features like the employed bee phase, the onlooker bee phase and the scout bee phase. A case study is considered to demonstrate the performance of the ABC algorithm for the LFTPVDS and compare the total cost-to-profit ratio obtained with the Joshi & Gupta (2011) approach. The same upper bound objective function value was found using both methods. However, the lower bound of the objective function value found using the ABC algorithm was lower than the objective function value found using the Joshi & Gupta (2011) approach. The total cost to total profit ratio is obtained using the ABC algorithm and reaches a point where the minimum total cost to profit ratio occurs. ABC algorithm performs better than the Algorithm proposed by Joshi & Gupta (2011) when solving LFTPVDS.

Keywords: Artificial Bee Colony algorithm, Fractional transportation problem, Varying demand and supply

COMPUTING MULTI-DIMENSIONAL PERSISTENT HOMOLOGY TO STUDY PROTEIN STRUCTURES

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Persistent Homology (PH) is the central tool of Topological Data Analysis. With PH, one can get a topological summary of data, which exists as a point cloud in a high-dimensional ambient space. Persistent homology has been effectively used in many areas of sciences, such as physiology, genomics, neuroscience, physics, and economics, to reveal hidden geometric structures in data that could help to expand the understanding of each field. In molecular biology and biochemistry, it is a well-known fact that the geometry of protein folding plays a crucial role in how the protein interacts with other molecules. In this research, we apply persistent landscape (PL), a summary of PH, which is conducive to analyzing the geometry of proteins. We analyzed the Protein Data Bank (PDB) files of various proteins using persistent landscape tools. By using PL, we were able to reduce computational time by reducing the number of points in the data cloud, performing the PH computation, and averaging the results. In our computations, we discovered that essential topological characteristics, like PH, could be computed using PL in less computational time. Furthermore, we found that the persistence diagrams and bottleneck distances, which are indicators of flexibility and stability, exhibited similar results when computed using Persistent Landscape (PL). The practical implications of our research are substantial. Understanding the geometry of protein folding and its relationship to molecular interactions is essential for advancing fields such as drug discovery, enzyme engineering, and protein design. By employing PL as a condensed summary of PH, researchers can rapidly analyse protein structures and make informed decisions regarding the development of therapeutics or the design of novel biomolecules.

Keywords: Flexibility, Persistent Homology, Persistent Landscape, Protein Classification, Topological Data Analysis

AN ALTERNATIVE APPROACH TO PROFIT MAXIMIZATION USING THE TRIANGULAR FUZZY TRANSPORTATION PROBLEM

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Optimization methods help resolve problems in the real world, such as project schedules, assignment problems, and network flow analysis. We cannot solve transportation problems effectively using traditional, classical techniques. It is more effective to solve real-world problems using fuzzy transportation problems. Transportation problems with uncertain supply, demand, or transportation expenses are known as fuzzy transportation problems. In this research, the maximum fuzzy transportation problem aims to identify an accurate and alternative method to allocate resources between destinations while maximizing overall profit, where fuzzy demand and fuzzy supply are all in the form of triangular fuzzy numbers. The literature attests that different techniques have been developed in the past to solve the triangular fuzzy transportation problem. Many researchers have introduced various alternative methods to find the initial basic feasible solution (IBFS) value for maximizing the overall profit of triangular fuzzy transportation problems. Accordingly, this research aims to present an alternative method that can obtain IBFS for transportation problems while maximizing fuzzy profit. The proposed method is based on the mean value of each column and row of the transportation cost/profit table using the crisp value obtained by Yager's robust ranking method. It has been shown how to obtain IBFS so that the profit from fuzzy transportation problems is maximized. Additionally, since the IBFS generated by our approach is discovered to be extremely close to the optimal solution, it can be inferred from the comparison of the solution with alternative approaches that this method is more efficient and less time-consuming than the existing methods. Additionally, problems in various other disciplines where data is in the form of triangular fuzzy numbers can be solved using this ranking technique.

Keywords: Fuzzy transportation problem, Initial basic feasible solution, Optimal solution, Profit maximization, Yager's ranking method

PATTERN GENERATION FOR TWO-DIMENSIONAL CUTTING STOCK PROBLEM

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The selection of feasible cutting patterns to minimize raw material wastage, known as the cutting stock problem, has become a key factor in success in today's competitive manufacturing industries. Cutting stock problems can appear in the form of one-dimensional and two-dimensional. Its objective is to determine the optimum layout of cutting items to improve the utilization ratio of raw material (main sheet), which minimizes the cut loss without overlapping the cutting items. The output obtained by solving the cutting stock problem can optimize the resources while meeting the given supply demand. This is achieved by minimizing wastage leading to a reduction in the cost of production. This study aims to modify the Branch and Bound Algorithm to generate cutting patterns for two-dimensional rectangular/square and circular shape cutting items. Our study is restricted to raw materials (main sheets) in a rectangular shape with known dimensions. Hypothetical data is used with four rectangular/square shape sets, and three circular shape items with fixed diameters are used to determine the cutting patterns by applying the modified Branch and Bound Algorithm and obtaining the optimum packing schedule using the developed Mathematical model. In this study, feasible cutting patterns are generated using the code programmed in the Matlab environment, and then Microsoft Excel is used to solve the developed mathematical model to generate the optimum packing schedule.

Keywords: Branch and Bound Algorithm, Pattern generation, Two-dimensional cutting stock problem, Matlab

ODD HARMONIOUS LABELLING OF LADDER GRAPHS

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The ladder graph is a graph that consists of two parallel lines connected by cross-lines and is a fundamental graph in the study of graph theory. In this research paper, we focus on the concept of odd harmonious labelling of the ladder graph. A graph G with q edges is called harmonious if there is an injective function between the set of vertices of G and the set of integers modulo q , and the edge labels should be distinct. A graph G is considered odd harmonious if there exists a unique way to assign each vertex of the graph a distinct even number between 0 and $(2q - 1)$, such that the sum of the assigned numbers for any two adjacent vertices results in a unique, odd number between 1 and $(2q - 1)$. The concept of odd harmonious labelling was first introduced by Rosenfeld in 1979 and has since been an active area of research in graph theory. In this research, we mainly focus on odd harmonious labelling. The ladder graph is one of the simplest and most well-studied graphs in graph theory, making it a suitable starting point for studying odd harmonious labelling. The ladder graph has a number of exciting properties that make it an ideal candidate for this research. For example, the ladder graph is a bipartite graph, which means that its vertices can be divided into two sets, such that each vertex in one set is only connected to vertices in the other set. Additionally, the ladder graph is a planar graph, meaning that it can be drawn on a flat surface without any of its edges intersecting.

Keywords: Harmonious labelling, Odd harmonious labelling, Ladder graph

**A MANUFACTURES AND BUYERS INTEGRATED PRODUCTION INVENTORY
SUPPLY CHAIN MODEL WITH UNEQUAL SIZE OF BATCH TRANSFER**

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Recent studies show that to retain in the competitive market; business consultants are more focused on optimizing the total expenses by controlling the supply chain management system. To address this current focal point, researchers have initiated relevant studies in a broader spectrum. The manufacturer and buyer are acting a significant role in two-echelon supply chain management systems. So, the manufacturer and buyer integration is more beneficial to both parties than without proper coordination. In the literature, many studies have been conducted more on single manufacturer and buyer(s) integration than multiple manufacturers and buyers scenarios. Therefore, this study develops a mathematical model for manufacturers' and buyers integrated production inventory systems and an efficient solution technique to minimize the joint total inventory costs of both parties. Manufacturers produce a single product at a finite production rate and distribute it to all buyers in a finite number of unequal size batches. In this study, it is assumed that batch sizes follow a geometric series and that manufacturers' production time of a batch is the same as the time needed to satisfy the buyers' demand by the previous batch. The inventory model developed in this study is validated by a case study. It was observed that the model optimizes the total inventory cost and reduces product prices while encouraging manufacturers and buyers to establish a closer relationship. Finally, the computational investigation shows that the integrated model provides significant cost savings to both parties by minimizing inventory costs.

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Keywords: Integrated production inventory, Manufactures and buyers, Unequal size of batches

MODELING THE TEA PRICES AT THE SRI LANKA TEA AUCTION: A TIME SERIES APPROACH

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The tea industry plays a significant role in the national economy of Sri Lanka. Currently, Sri Lanka ranks as the fourth largest tea producer in the world. On the global export front, the country holds the position of the third-largest exporter of tea. Since the tea auction market in Sri Lanka is mature, the leading tea-producing countries such as China, India, Indonesia, and Kenya are concerned about Sri Lankan Tea sold through the Sri Lanka tea auction. This study was conducted to analyses and model the weekly tea auction prices at the Sri Lanka tea auction based on time series analysis. The secondary data of weekly tea auction prices from 2011 to 2020 was collected from the official website of the Sri Lanka Tea Board used for this study. The analysis is done by using R software. This paper employs an asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) model is used to investigate the volatility and model the volatility on the tea prices at the Sri Lanka tea auction. Preliminary analysis indicated that the average price at the Sri Lanka tea auction is Rs. 482.98 per kg. Then, the stationarity is checked using the augmented Dickey–Fuller (ADF) test, and the conditional mean model AR(1) model is fitted to differenced series. After checking the residuals of the AR(1) model, conditional heteroscedasticity in residuals of the fitted AR(1) model is detected using the ARCH LM test. Under this effect, the conditional variance model GARCH(1,1) was built to forecast the tea prices. Thereafter, the asymmetric effect was checked using the sign bias test, and some asymmetric GARCH models were considered. Finally, the AR(1)-GARCH(1,1) is selected as the best for modelling tea prices by comparing Bayesian Information Criteria (BIC) and checking the parameters and residual of the model. The accuracy of the model is measured using MAPE. MAPE of this model is 10.09%. This model is the capacity to capture the volatility of the weekly tea auction prices at the Colombo tea auction.

Keywords: ARCH, GARCH, Forecasting, Tea Auction, Time Series

**UNBALANCED MAXIMIZATION ASSIGNMENT PROBLEM
WITH MULTIPLE ASSIGNMENTS**

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An assignment problem is a particular case of a transportation problem where the objective is to assign several resources to an equal number of activities to minimize the total cost or to maximize the total profit of allocation. Hungarian Algorithm is applied to solve the assignment problem with minimization or maximization problems. When the Hungarian Algorithm is applied to solve an unbalanced assignment problem in which the number of jobs and the number of machines is not equal, the procedure assigns some of the jobs to dummy machines or some of the machines to dummy jobs, ignoring the execution of those jobs. So, it is suggested to perform each job which can be conducted by assigning more than one job to a single machine rather than assigning the excess jobs to dummy machines. In this study, modified Dhouib-Matrix-AP2 is proposed to solve the Unbalanced Maximization Assignment Problem without converting the problem to a minimization problem. The assignment matrix is modified by adding the maximal processing efficiency for each column or row (to make the matrix balanced) and then applying the modified Dhouib-Matrix-AP2 to acquire the optimal assignment. A hypothetical example with seven jobs and five machines is used to compare the Hungarian Algorithm with the proposed modified Dhouib-Matrix-AP2. The Hungarian Algorithm was applied to solve an unbalanced maximization assignment problem, resulting in a maximum objective function value of 270. A modified DM-AP2 method developed with Python was used to obtain a result of 321. This method was verified with Excel Solver.

Keywords: Assignment problem, Dhouib-Matrix-AP2, Unbalanced assignment problems

**IMPROVING THE PRODUCTION EFFICIENCY IN APPAREL INDUSTRY
THROUGH STATISTICAL PROCESS CONTROL: A CASE STUDY**

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The apparel industry has a significant impact on the Sri Lankan economy. The main activities in the process of the apparel industry are cutting fabrics and sewing them together using threads to produce apparel. In the production process of an apparel production plant, quality control is the most significant factor in the successful productivity of apparel. The principal objective of this study is to identify the current malpractices of quality control techniques and introduce more effective Statistical Process Control (SPC) techniques to a well-known apparel industry known as Compass International Pvt. Ltd. Due to practical difficulties in collecting data, only the defective items per hour and the total number of items produced per hour were collected for our study. After analyzing the data, it was decided to introduce Fraction Nonconforming Attribute Control Chart with a variable sample size. The control chart (CC) was implemented using three alternative methods: variable width control limits, control limits based on the average sample size and the standardized CC. A critical analysis was performed to diagnose the cause of more frequently occurring defects. Accordingly, diagnosed key factors for the out of control in quality were due to various human factors and machine failures. Absenteeism and turnover of operators, less skilled operators being assigned in the production process and poor physical and mental health condition of operators contributed to more defects in the finished items. Improper machine adjustments for the specific operation had caused to produce defective products. Also, frequent breakdowns of the sewing machines led to uneven thread tension, slip stitch, broken stitch, oil marks etc. In conclusion, this study recommends the management adopt SPC and also to take appropriate recovery measures to improve the quality of the production process.

Keywords: Apparel industry, Control chart, Statistical process control

SKOLEM AND HOOKED SKOLEM SEQUENCES UPTO ORDER EIGHT

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A Skolem sequence of order n is a sequence of $2n$ integers consisting of pairs of positive integers from 1 to n with the property that the two copies of $k \in \{1, 2, \dots, n\}$ appear exactly k positions apart. In particular, a Skolem sequence of order n is a sequence $S = \{s_1, s_2, \dots, s_{2n}\}$ of $2n$ integers such that; for every $k \in \{1, 2, \dots, n\}$, there exist exactly two elements $s_i, s_j \in S$ such that $s_i = s_j = k$, and if $s_i = s_j = k$ with $i < j$, then $j - i = k$. On the other hand, a hooked Skolem sequence is a sequence of $2n + 1$ integers with the additional property that the $2n^{th}$ integer is null. Skolem sequences originated as a result of the studies done by Thoralf Albert Skolem in 1957 to find a solution for Heffter's difference problems. Later, many studies were done on these sequences, which are now used in various applications: Abraham (1991) used Skolem sequences in labeling of graphs, Colbourn and Rosa (1999) in construction of triple systems, Pike and Shalaby (2000) in factorization of complete graphs. Even though there are papers that address how many sequences there are for various Skolem-type sequences, up to now, there have not been any that give the exact sequences. In this paper, we give Skolem and hooked Skolem sequences up to order eight so that they can be directly used in the applications. Also, we present a theorem which helps write Skolem sequences. Hence, that theorem can be used to develop a computer program to find Skolem sequences.

Keywords: Graph labeling, Heffter's difference problems, Hooked Skolem sequences, Skolem sequences, Triple systems

ALEXANDROV UNIFORMITY OF INVERSE SEMIGROUPS

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It is well-known that a topological abelian group admits a canonical uniform structure, and this has many applications in commutative algebra. For example, the algebraic completion of a commutative ring R with respect to a filtration $R \supset I \supset I^2 \supset \dots$ generated by a maximal ideal I can be thought of as the Cauchy completion with respect to the I -adic uniform structure on R whose entourages are generated by sets of the form $E_n = \{(a, b) \in R \times R \mid a - b \in I^n\}$. Accordingly, it is natural to question the existence of uniformities on algebraic structures other than groups. On the other hand, it is recently proven that the set of certain equivalence classes of uniform structures on double (disjoint union with itself) of a uniform space form an inverse semigroup. Likewise, there are many glimmerings in the literature that hint at some duality between inverse semigroups and uniform spaces. In this project, we add another piece of evidence to this list by constructing a non-Archimedean uniform structure on an inverse semigroup stemming from the Alexandrov topology induced by its natural partial ordering. This construction defines a functor from the category of inverse semigroups and their homomorphism to that of uniform spaces and uniformly continuous functions between them.

Keywords: Topological Inverse Semigroup, Uniform Spaces, Alexandrov Topology

OPERATOR NORM OF FUNCTIONALS ON SUBSPACES OF HARDY SPACE OF THE UNIT DISK ($H^2(D)$) AND CONSEQUENCES

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The Hardy space of the unit disk is the space of all holomorphic functions on the open unit disk (D) such that the coefficients of the power series representation of functions are absolutely square summable. For $T \in H^2(D)^*$, by Riesz representation theorem, there is $g \in H^2(D)$ such that $T(f) = \langle f, g \rangle$ for all $f \in H^2(D)$. Moreover $\|T\|_{op} = \|g\|$. $\|\cdot\|$ is induced by the inner product. Let A be a closed subspace of $H^2(D)$. It is interesting to know the operator norm when T is restricted on A . $\|T_A\|_{op} = \|Pg\|$ on A , where $P: H^2(D) \rightarrow A$ is the orthogonal projection and T_A is the restriction of T on A . For special types of subspaces of the Hardy space, we can use kernel functions to study the operator norm of restrictions of the point evaluation. With this work, we shall recover some standard results in Complex Analysis. Let $S_n = \{x_1, x_2, \dots, x_n\} \subset D$ of distinct elements. Define $H_{S_n} = \{f \in H^2(D) : f(S_n) = 0\}$. The space H_{S_n} is a closed subspace of $H^2(D)$. For $z \in D$, define $B_n(z) = \prod_{i=1}^n \frac{x_i - z}{1 - \bar{z}x_i}$. The function B_n is an inner function and the multiplication operator $M_{B_n}: H^2(D) \rightarrow H_{S_n}$ defines an isometry isomorphism, from which it follows from RKHS theory that $k_z^{S_n}$ of H_{S_n} at a point z is given by $k_z^{S_n}(\omega) = \frac{B_n(z)B_n(\omega)}{1 - \bar{z}\omega}$. We shall provide a simple proof for this result using Mathematical induction and show that $\|T_z^{S_n}\| = \frac{1}{\sqrt{1-|z|^2}} \prod_{i=1}^n \frac{|z-x_i|}{|1-\bar{z}x_i|}$ on H_{S_n} where $T_z^{S_n}: H_{S_n} \rightarrow \mathbb{C}$ is the point evaluation at $z \in D$. With this work, we recover a version of the identity theorem in Complex Analysis. That is, a holomorphic function $f: D \rightarrow \mathbb{C}$ which becomes zero on a subset S' of D with a limit point in D , is identically zero on D . We shall generalise this idea to find the Kernel functions on Subspaces of the Bergman space of the unit disk, more generally on weighted Hardy spaces.

Keywords: Hardy Space, Holomorphic function, Identity Theorem

**HOMOTOPY ANALYSIS METHOD FOR NONLINEAR SINGULAR BOUNDARY
VALUE PROBLEMS OCCURRING IN VARIOUS PHYSICAL SITUATIONS**

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In this study, we focus on finding analytical solutions to nonlinear singular boundary value problems (SBVPs) arising in various physical phenomena using the Optimal Homotopy Analysis Method (OHAM). We also present several numerical examples from physical situations to demonstrate the efficacy of the technique. The Optimal Homotopy Analysis Method can be used to obtain series solutions for nonlinear coupled systems. Usually, if the operator of a nonlinear differential equation contains a linear part and then a nonlinear part with a parameter, the original nonlinear problem can be transformed into an infinite number of linear sub-problems. However, the OHAM solution process contains an auxiliary parameter that provides a convenient way of controlling the convergence of the series solution. Comparisons are made with available results in the literature for some significant cases to validate the analytical method, and the results are in good agreement.

Keywords: Homotopy Analysis Method (HAM), Singular boundary value problems (SBVPs)

**FINITE DIFFERENCE METHOD TO SOLVE ONE-DIMENSIONAL
SHALLOW WATER EQUATIONS**

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This study investigates the numerical solution of the nonlinear system of partial differential equations describing the one-dimensional shallow water equations. The explicit scheme of finite difference methods for the discretization of space and time variables has been used to solve this system of equations with initial and boundary conditions. The shallow water equations are hyperbolic partial differential equations that describe the fluid flow in a layer of constant density with a horizontal scale greater than the layer depth. The governing equations are derived from the conservation of mass and linear momentum and are commonly used for modeling environmental fluid dynamics. In particular, the flow of water in a wide channel with a rectangular cross-section and a smoothly varying bottom surface is governed by the one-dimensional shallow water equations. To solve the nonlinear partial differential equations, the discretization process is employed for the linearization of the set of equations. The resulting system of equations is then solved using matrix representations and inverse matrix methods. The numerical experiments are conducted using MATLAB software, and the obtained results are presented and discussed.

Keywords: Explicit scheme, Finite difference method, Numerical solutions methods, One-dimensional shallow water equations

INCIDENCE COLOURING OF STAR GRAPHS

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The star graph of order n , denoted, S_n is a simple graph with n vertices, one distinguished vertex of degree $n - 1$ and the remaining vertices of degree 1 and adjacent to the distinguished vertex. A colouring of occurrences is a mapping from integers to a set of colours in which no two consecutive incidences of G have the same colour. Then, an incidence colouring of G assigns a colour to each incidence of G in such a way that adjacent incidences receive distinct colours. The incidence chromatic number of G is denoted by $\chi_i(G)$, and it is the smallest number of colours required for incidence colouring. Brualdi and Massey introduced the concept of a graph's incidence chromatic number (1993). In this paper, we focus on the incidence chromatic number of star graphs and give the incidence chromatic number of the infinite star graph family. The star graph is one of the most basic and well-studied graphs in graph theory, which makes it an excellent place to start when learning about incidence colouring and incidence chromatic numbers. The star graph has several intriguing properties that make it an excellent candidate for this study. The star graph, for example, is a complete bipartite graph, which means that its vertices can be divided into two sets, with each vertex in one set only connected to vertices in the other set. Furthermore, the star graph is a planar graph, which means that it can be drawn on a flat surface without any of its edges intersecting.

Keywords: Incidence colouring, Incidence chromatic number, Star graph

A NOVEL CRYPTOGRAPHIC SCHEME USING THE SUPER MAGIC COVERING OF WHEEL GRAPHS

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Cryptography is the study of techniques to ensure information security. Among the mathematical tools used in implementing Cryptographic schemes, Graph theoretic concepts get their prominence. In graph labeling, each vertex or edge of a graph is assigned a label or a value, which can be used to encode information and graph labeling can be used to design cryptographic protocols that rely on graph-based structures. Since numerous graph labeling techniques are available in the literature, it is perfectly safe to use graphs in cryptography as it is difficult to locate the labeling technique and distinguish between the graphs used for encryption. In 2020, M. Giridaran used super magic labeling in encryption. Taking that as the base, in the present cryptographic scheme, we use graphs with snake vertex labeling to encrypt a plaintext symmetrically using the randomness of the distribution of super magic covering of wheel graphs of odd order with an advanced shift cypher. We have proposed a new cryptographic scheme in which the sender and the receiver hold the secret key pair(k, l), which has been chosen appropriately. A plaintext is encrypted using an odd number k , which is the number of sides of a cyclic graph (polygon), and l is chosen to preserve the symmetries of consecutive snake graphs. Graph labeling offers an effective tool for creating flexible, efficient, and secure cryptographic protocols. We have created a protocol that is specialized for encryption and offers a high level of security by taking advantage of the structure and the randomness of the labeling of graphs. By carefully selecting the labeling scheme, we have designed a protocol that is secure against various types of attacks, such as impersonation and replay attacks.

Keywords: Shift cypher, Snake vertex labeling, Super magic covering, Wheel graph.

Mathematics Education

**MATHEMATICS TEACHERS' PERCEPTIONS OF FLIPPED CLASSROOM
TOWARDS 21ST-CENTURY SKILLS**

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The field of education has experienced significant advancements, particularly in the learning-teaching process, due to the rapid development of digital literacy in the 21st-century. Teachers should be aware of new technological teaching methods while addressing 21st-century competencies since they are the pioneers in guiding students toward our nation's future. One popular pedagogical practice in the field of education, known as the flipped classroom approach (FC), offers students the opportunity to engage in self-learning experiences in a digital era. The main purpose of this study was to find out the mathematics teachers' perceptions of FC towards 21st-century skills. The survey research design was used and the questionnaires were administered to 244 mathematics teachers from the Galle Education Division using the purposive sampling method. Data were analyzed quantitatively using the SPSS (Version 25) software. Face-to-face interviews were conducted with 10 randomly selected mathematics teachers from the sample for data triangulation. It was revealed that 75.8% of teachers were in the view that the FC approach can monitor students' creativity when learning mathematics. In addition, 82.1% of teachers stated that FC can promote students' communication skills. Furthermore, 75% of the sample agreed that this approach provides critical thinking skills by sharing knowledge with their peers through smart computing and as students acquire a large amount of information through technology and engage in critical thinking before entering the classroom. Also, 81.7% of the teachers agreed FC enhanced face-to-face discussions for actual collaborative applications for students. The findings also revealed no statistically significant differences (at $\alpha \leq 0.05$) on the total degree of perceptions of FC towards the 21st-century skills and the gender ($t(df) = -0.491$, $p = 0.125$). The results demonstrated that the teachers were well aware of the importance of the FC approach. The interview results of the study revealed that weak internet connection, lack of knowledge of social platforms, limited resources, insufficient technological tools and inadequate teacher training, workshops and refresher courses related to the FC approach discouraged implementing the FC approach in schools in Sri Lanka. Consequently, it is recommended to employ practice-oriented strategies to enhance the implementation of FC among mathematics teachers in Sri Lanka.

Keywords: Mathematics teacher, Flipped Classroom, Perceptions

EXPLORING MATHEMATICAL LANGUAGE OF TEACHERS IN TEACHING CONCEPT OF LIMIT

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The concept of limit aggregates an enormous number of new symbols, notations and vocabulary to students' language repository. This study explores the use of mathematical language in teaching 'limits' for advanced level (A/L) students. Data was drawn from a two-year extensive study that investigates the Mathematical Knowledge for Teaching (MKT) of secondary level mathematics teachers in teaching limits and derivatives. Fifteen A/L mathematics teachers across three districts in Sri Lanka participated in the study, whose lessons were observed, video recorded and transcribed. The data was qualitatively analyzed through the lens of MKT framework under two main constructs: symbolic expressions and natural language. The analysis revealed several strengths and weaknesses in teachers' use of mathematical language. The majority of the teachers explicitly introduced symbols and vocabulary by critically discussing its usage and meaning. However, inconsistencies were noticed among some teachers with respect to the spatial orientation, relative font size and relative position of symbolic representations. Some teachers found it challenging to adapt to the correct usage of the syntax for the notation of limit and mostly read the notation as the way it was written. Misconceptions were also noticed when converting abstract ideas into concrete form. In fact, some teachers borrowed words from their colloquial register to give an intuition of the mathematical word "Limit". Several flaws were noticed due to slip of the tongue, inconsistencies in the use of phrases, ambiguous language with demonstrative pronouns like *this*, *that*, and *these*. These results shed light into the importance of mathematical language in delivering complex mathematical ideas. Teachers need to be mindful with extemporaneous speech during instruction. This study informs teacher development programs, workshops and teacher education courses to place an emphasis on inculcating better mathematical language habits among teachers. This will ensure consistent and precise language usage when mathematical ideas are communicated.

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Keywords: Limit of a function, MKT framework, Teaching secondary mathematics

**EFFECTIVENESS OF MOTIVATIONAL STRATEGIES
ON PERFORMANCE IN MATHEMATICS OF GRADE SIX STUDENTS**

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Mathematics is a vital subject that has been highly valued and highly regarded since ancient times. Nowadays there is hardly any education without the knowledge of mathematics. For children, moving from primary to junior secondary level in mathematics is challenging and stressful. The aim of the present study is to use motivational strategies in grade six mathematics to enhance students' performance. This was an action research study. The convenience sample consisted of 22 students of grade six. Both quantitative and qualitative data were gathered by using performance test papers, questionnaires, observations, and informal discussions. Descriptive statistical analysis like mean and standard deviation were used to analyze quantitative data while thematic analysis was used with qualitative data. The Mean Value (MV) and the Standard Deviation (SD) for the mathematics performance test were 29.2 and 21.47, respectively before the intervention. The first intervention was to enhance mathematics performance by developing self-motivation and intrinsic motivation of students, and their performance in mathematics was MV=41.0 (SD=17.9) at the end of the first cycle. The second intervention was to improve students' self-regulated learning skills in mathematics which was measured as MV=48.5 (SD=16.5) while the third intervention was to enhance performance through self-efficacy, and was measured as MV=53.5 (SD=12.4). The last intervention of the action research was used to avoid test anxiety of students where mathematics performance of students was measured as MV=60.8 (SD=14.0) at the end. The qualitative findings revealed that activities related to motivational strategies have improved enthusiasm and fulfilled the different needs of students related to mathematics. Also, it was observed that different motivational strategies will improve the performance of mathematics of students in Grade six. Moreover, the avoidance of test anxiety affects better contribution towards students' performance in mathematics. In conclusion, we recommend using different motivational aspects while recognizing students' situation to enhance their performance of mathematics in Grade six.

Keywords: Self-motivation, Intrinsic motivation, Self-regulation, Self-efficacy, Test anxiety

ACTIVITY-BASED APPROACH IN TEACHING GEOMETRY FOR LOW ACHIEVING STUDENTS IN GRADE TEN

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In mathematics education, students often struggle to understand abstract geometrical concepts. In the 21st century, traditional teaching methods are no longer sufficient for the active learning processes that are required by the constructivist learning approach. The aim of this study was to investigate the effectiveness of an activity-based approach in teaching mathematics to grade 10 students who were low achievers and to suggest ways to enhance their performance. This action research study involved a purposive sample of 18, grade 10 students who were struggling in mathematics. Both quantitative and qualitative data were collected using mathematics performance tests and a questionnaire. Four interventions were implemented to enhance the students' performance: traditional teaching methods, classroom activities using nail boards and colored papers, technology-based activities using GeoGebra software, and blended learning using hands-on classroom practices and computer-based activities with GeoGebra software. The performance of the students was evaluated using descriptive statistical methods, and qualitative data were analyzed thematically. The results showed that the students' performance improved gradually through the intervention process, with mean values of 25.05, 46.83, 56.27, and 73.05 in the first, second, third, and fourth interventions respectively. The qualitative findings indicated that the interventions increased the students' motivation and enthusiasm for learning geometry. The findings suggest that hands-on activities and computer-based practices can be effective in enhancing student performance and motivation for meaningful learning of grade 10 geometry. The results support the use of an activity-based approach to teaching mathematics, particularly for students who struggle with abstract concepts. In conclusion, this study highlights the importance of adopting innovative teaching methods to improve students' learning outcomes and prepare them for the challenges of the 21st century.

Keywords: Activity-based learning, Blended learning, Constructivist learning approach, GeoGebra, Geometry

**OVERCOMING BEHAVIOURAL PROBLEMS OF A GRADE EIGHT STUDENT:
AN ACTION RESEARCH**

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Action research is a powerful tool to solve problems in the field of education by implementing appropriate solutions to the existing challenges in order to achieve organizational goals. This study was based on one of the students' behavioral problems such as pumping water to students, disturbing class by making noise, walking around during the lesson, lack of concentration during the lesson, hitting students and kicking the door of a washroom when there was another student inside, squeezing the neck of students and throwing stones to junior students. This student's performance in mathematics was also unsatisfactory due to the above behavioral problems. Hence, he was taken to a satisfactory level by appointing him as the monitor, assigning several leadership tasks and maintaining psychological relatedness in the intervention process. Data were collected for a period of six months from teacher interviews, classroom observations, watching CCTV cameras, and parent interviews. The collected data were analyzed using thematic analysis. Based on the interventions carried out, the child's behavior was improved gradually and student performance of mathematics was also improved a lot. The themes generated in this study include: positive behavior through acceptance; development of personality by leadership roles; and enhancement of enthusiasm as well as confidence due to self-satisfaction. Thus, children's behavior could be changed through psychological relatedness, autonomy support, and enhancement of competencies through suitable interventions. The outcome of this study could be used to encourage other teachers and educators to facilitate their students to change their behavior positively in order to prepare efficient and effective citizens for the world.

Keywords: Action research, Behaviour, Interventions, Development of confidence

**FACTORS ASSOCIATING WITH G.C.E. (O/L) EXAMINATION FAILURES IN
WESTERN PROVINCE**

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General Certificate of Education (Ordinary Level), G.C.E. (O/L), is an examination conducted by the Department of Examination, Sri Lanka, and is an important examination for school children. Passing this examination is a minimal requirement for further education or formal employment in the public or private sector. Therefore, the G.C.E. (O/L) examination is a critical barrier for Sri Lankan school children. However, around 25% of the students who sit the examination fail every year. In order to improve the passing rate, workshops are conducted by the Ministry of Education, Provincial Education Departments, Zonal Education offices and schools. Every year, the government invests a considerable amount of money to remedy the problem. However, the outcome of these efforts are not effective in improving the G.C.E. (O/L) results, as these programs may not target the real issues at ground level. Thus, the aim of the study was to investigate the factors associated with G.C.E. (O/L) failures in the Western Province so as to make appropriate suggestions to remedy the situation. This was a quantitative study and it consisted of a sample of 5,101 students from 55 schools who failed G.C.E. (O/L) examination. Data was gathered from the Examination Department. Microsoft Excel and SPSS were employed in the data analysis. Results revealed that the highest number of students failed due to the failure in Mathematics. Only a few students had the issue of not passing the first language in G.C.E. (O/L). Hence, passing Mathematics in G.C.E. (O/L) is a valuable determinant to pass G.C.E. (O/L) examination. Thus, appropriate remedial measures are to be found and implemented to facilitate students following G.C.E. (O/L) to secure their future.

Keywords: Mathematics, G.C.E. (O/L), Quantitative study

**RELATIONSHIP BETWEEN PRIMARY TEACHERS' PEDAGOGICAL
CONTENT KNOWLEDGE AND STUDENTS' ACHIEVEMENT IN
MATHEMATICS**

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Education should empower individuals to navigate future challenges confidently while fostering technological proficiency for enhanced ease of life. Among all subjects, Mathematics plays a pivotal role, permeating various aspects of our daily existence. In the Sri Lankan Education system, Mathematics Education starts from grade 1. However, after completing 11 years of compulsory education, the students' performance in Mathematics is not satisfactory. Primary education should provide the learner with the desire to continue learning, develop the ability for critical thinking and logical judgment, appreciate and respect the dignity of work, and develop desirable social standards. Thus, teaching Mathematics in primary education provides opportunities for developing important intellectual skills in problem-solving, deductive and inductive reasoning, creative thinking and communication. The teacher is an important and effective character in primary education determining students' success in developing Mathematical background and its achievement. In order to accomplish the objectives, primary teachers must possess proficient pedagogical content knowledge. In the literature review, it was revealed that pupils' failure of Mathematics is mainly due to pitfalls in teaching. Thus, this study aimed at finding out the relationship between Primary Teachers' Pedagogical content knowledge and student achievement in Mathematics. The specific objectives of this study were to assess the levels of pedagogical content knowledge among primary teachers, determine the achievement levels of stage one students in Mathematics, investigate the correlation between primary teachers' pedagogical content knowledge and student achievement in Mathematics, and provide suggestions for enhancing pedagogical content knowledge to improve the academic performance of grade one students in Mathematics. Through convenience sampling, 64 teachers in grade 2 classes and 256 students from four zones in the Kurunegala district were selected. Quantitative as well as qualitative data were collected through questionnaires, tests, interviews, document search and observations. Data triangulation was conducted to increase the validity of the findings. The results showed that the teachers' content knowledge was around 70%. All Mathematical concepts were covered by the given test paper and the achievement level of "pre-mathematical concepts" of the teachers was up to the level of 63%. Moreover, primary teachers' content knowledge showed a significant positive correlation with stage one student achievement with a correlation coefficient of 0.65. Thus, it is apparent that teachers' content knowledge has a significant impact on students' achievements in Mathematics. Hence, it is imperative to improve the content knowledge of primary teachers in order to enhance student performance.

Keywords: Assessments, Content knowledge, Evaluation, Pedagogical Content knowledge

**IMPROVING THE MATHEMATICS RELATED VOCABULARY USING
BILINGUAL GLOSSARIES: AN ACTION RESEARCH IN A 1AB MULTIETHNIC
SCHOOL IN CENTRAL PROVINCE**

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Through the experience as a bilingual teacher since 2006, It was identified that there is an inter-relationship between knowing the mathematics technical terms in the students' first language and them in the less familiar targeting language of English (L2). Mathematics is an Abstract subject that is essential in daily activities. This research was conducted in a Muslim Girl's College in the hill capital with a multicultural environment in bilingual classrooms. This action research aims to improve the Mathematical related vocabulary skills of the participants who were studied in grade 07 at the time when the research was conducted with three research questions. All the thirty students in the grade 07 bilingual class whose first language was Tamil were used to conduct this Action Research. The quantitative method was used to conduct this research and simple statistics were used to analyze data. The participants were asked to prepare a list of new words and by using those words prepared the Mathematics Calendar, Mathematics Glossary, and Puzzles to find the relevant technical terms. The achievement of the participants was measured by using two different papers. The biggest challenge that was faced while engaging in the teaching-learning process was that the participant's first language(L1) was Tamil while the researcher's L1 was Sinhala. It was identified that the positive correlation between the student's first language(L1) and the subject of Mathematics increased after conducting this action research. At the same time, the results matched with the Jimm Cummings Common Underlying Proficiency. It is recommended to use both textbooks in English and L1.

Keywords: Action research, Bilingual Education, Glossary, Vocabulary

THE USE OF GEOGEBRA IN TEACHING AND LEARNING CONCEPTS IN GEOMETRY

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Teachers frequently face various difficulties in their efforts in developing geometric concepts in the minds of students. In this case, it is no secret that the concepts related to geometry are considered to be the most abstract and difficult to grasp among all other mathematical concepts. In the literature, there were numerous studies aiming at this issue. Among various approaches, many educators have attempted to teach the geometrical concepts to the students with the help of modern technology, and eventually gained positive results. This study focuses on using GeoGebra software to increase the motivation of students to understand concepts in geometry. The study sample was 69 students of grade 11 of a 1AB type school in the Denuwara Education Zone in Kandy district. The sample was randomly divided into two samples based on cluster sampling technique and labeled as control and experimental groups. Two theorems from “Cyclic Quadrilaterals” lesson were taught for both groups, for the control group using traditional teaching methods and the experimental group using the GeoGebra software. After conducting lesson units for 10 periods for each group, a test was administered to evaluate the students’ performance. Results revealed a significant learning improvement of the experimental group (9.50) compared to the control group (7.82) in grasping the concepts in geometry and also a clear interest was developed among the students in learning the subject with the help of GeoGebra. It was quite evident from the behavior of the students in the experimental group that not only they appreciated the new teaching method but also they became self-learners. In addition, when looking at the student’s motivation to solve geometry problems, it was observed that there was a very good student motivation. Also, it was revealed that 90% of mathematics teachers in the region do not use computer technology to teach geometry. The study concludes that bringing modern technology as a teaching tool will not only motivate students' attitude in learning the subject, but also mathematics curriculum developers to revise the curriculum in order to introduce effective and novel teaching methodologies.

Keywords: Cyclic Quadrilaterals, GeoGebra, Geometry

**TECHNIQUES TO ENHANCE GEOMETRICAL SKILLS OF SLOW LEARNERS:
AN ACTION RESEARCH OF GRADE 8 MONOLINGUAL STUDENTS IN A
SELECTED SCHOOL IN COLOMBO**

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Mathematics is both abstract as well as a concrete subject which is easy to understand when the concepts are related to the environment and day-to-day activities. Moreover, mathematics is a compulsory subject that is essential to pass the G.C.E. Ordinary Level examination. The aim of this action research is to use different techniques when teaching Geometry to grade 8 monolingual students and to make suggestions to enhance the performance of slow learners. The specific objectives of this action research are identifying slow learners, identifying different techniques that can be used to teach slow learners, to investigate the effectiveness of the techniques used to enhance the participants' basic geometrical skills and to make suggestions to support slow learners to enhance their geometrical skills. The purposive sampling technique was used when selecting the sample where 8 students were selected who scored less than 10 marks for the mathematics paper in the third term test from 40 monolingual students in the class. Test papers and questionnaires were used as the research instruments. Simple statistics and SPSS were used to analyze quantitative data. This action research included three interventions. In the first intervention, the participants were instructed to use textbooks, extra reading materials and the internet to find information on the given geometric sections. After presenting the findings and conducting a classroom discussion the participants were evaluated by giving a post-test paper on all the presented geometrical concepts. After reflecting the results, the second intervention where students engaged in hands-on activities on different theorems related to triangles and quadrilaterals was implemented. After conducting a post-test and reflecting on the marks, the third intervention was implemented. There, students were asked to teach the geometrical concepts taught in the first two cycles using the GeoGebra software. When comparing the post-test marks given at the end of the first two cycles, it was revealed that there was an improvement in the marks obtained by the participants, and they were motivated to engage in geometrical questions at the year-end examination. At the same time, it was very clear that the same technique was not effective for all the slow learners.

Keywords: Geometry, Monolingual, Techniques,

FACTORS AFFECTING G.C.E. (ORDINARY LEVEL) FAILURES IN WESTERN PROVINCE, SRI LANKA

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General Certificate of Education (Ordinary Level), G.C.E. (O/L), is an important examination for school students in Sri Lanka. Obtaining the G.C.E. (O/L) certificate with good results is a requirement either for further education or formal employment. However, approximately 25% of the students do not reach this crucial qualification, annually. The Ministry of Education together with other responsible authorities have conducted different programs and strategies to remedy this situation. However, despite these efforts, the increase of student performance is a challenging task. Thus, the aim of the study is to investigate the factors associated with G.C.E. (O/L) failures in the Western Province. The study was guided by two specific objectives such as (i) examine the awareness of G.C.E. (O/L) passing criteria among the education directors, principals and teachers and (ii) explore the remedial measures taken by each party to enhance G.C.E. (O/L) pass percentage in the Western Province. It was a qualitative study involving 34 participants including seven Deputy Directors of Education (Education Development), principals, Grade 11 class teachers, Mathematics teachers, and six first language teachers. Data collection was conducted through questionnaires and interviews. Content and thematic analyses were used in data analysis. Findings revealed that 43%, 57%, 71%, 0%, and 50% of directors, principals, class teachers, Mathematics teachers and first language teachers are not aware of all four criteria for passing G.C.E. (O/L) examination, respectively. Subject teachers proposed to implement remedial teaching programs at schools to enhance student performance in the G.C.E. (O/L). Thus, appropriate remedial measures are to be found and implemented to facilitate G.C.E. (O/L) students. Qualitative findings indicate themes such as conducting awareness programs for students, finding students' difficulties and issues in early stages, guiding the students to overcome their barriers, and implementing examination-oriented programs. Thus, the Grade 11 students have to be facilitated to enhance passing rate in their G.C.E. (O/L) examination in order to facilitate their aspirations for a better future.

Keywords: Awareness, Mathematics, G.C.E. (O/L), Passing rate, Remedial measures

**EFFECTIVENESS OF USING IQ QUESTIONS IN ENHANCING
PERFORMANCE OF GRADE SIX STUDENTS IN MATHEMATICS: AN ACTION
RESEARCH**

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Mathematics enhances the logical and rational thinking of people. Thus, mathematics education is highly valuable and important in empowering students' critical and logical reasoning. However, the performance in mathematics of some students is considerably low in some mathematics sums especially, in problems in wording. Thus, the aim of this action research is to guide students to use logical intelligence in solving problems given in the word format. As such, a test paper with two parts was prepared to administer among Grade 6 students. The sums in the first part consisted of questions to be solved directly using the basic mathematical operations and the problems in the second part consisted of mathematical questions with some details based on day-to-day activities. The results showed that six students who had high marks in the first part obtained low marks in the second part in the test paper, and they were selected for this study. These selected students were offered IQ (Intelligence Quotient) related questions and requested to work on them individually, and later were asked to explain how they reached their answers followed by a whole group discussion. At the end of a total of 15 hours of activities conducted three days a week, their progress was tested again by giving a similar question paper as at the beginning. The data were collected quantitatively and qualitatively followed by analyses through descriptive and thematic analysis. Following the intervention, approximately equal or higher marks were obtained for both sections by the students. Moreover, students' interest and enthusiasm were more than usual in the learning process in the classroom. Hence, logical intelligence is very important in learning mathematics as it is evident from the actions of those students. The IQ related activities motivate them to use such actions in their day-to-day life correctly and efficiently in addition to their technical knowledge. As mathematics is a measure of the intelligence of students. Instead of passing mathematics through rote learning, it is clear that the level of achievement could be easily enhanced by making the child accustomed to logical thinking using IQ related activities.

Keywords: Intelligence quotient questions, Intervention, Logical intelligence, Rote learning

ACHIEVEMENT OF COMPETENCY IN MATHEMATICS BY GRADE FIVE STUDENTS: A CASE STUDY IN PATHAHEWAHETA EDUCATION DIVISION, CENTRAL PROVINCE, SRI LANKA

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The Ministry of Education together with the National Education Research and Evaluation Centre carried out a national level evaluation on learning outcomes of Grade 5 students in 2019. The main objective of the evaluation was to inquire about the preparedness of the students to cope with the international evaluation system. The research reveals that outcomes of the two subjects, Mathematics and English, remain at 22.14% in Kandy which is below that of the national level, 24.18%. Moreover, it is revealed that the present Grade 5 students who were in Grade 3 in 2020 also have not met the expected learning outcomes of the two subjects. Thus, this study was aimed at investigating the achievement of competency in Mathematics by Grade 5 students' in Pathahewaheta Education Division in the Central Province, Sri Lanka. The Study sample consisted of 27 Sinhala medium schools and 23 Tamil medium schools including 750 Sinhala medium students and 250 Tamil medium students. A question paper was constructed and it was piloted with 50 other students. The paper was revised according to the answers and responses of participants. The paper was then administered to the student sample of the Pathahewahata division. Data were analyzed through Excel and SPSS. Results revealed that 19.5% students of Pathahewahata division have achieved expected competencies in Mathematics obtaining more than 90 marks. Around 37.3% of the students have achieved marks in the range of 70-90. However, 43.2% students have not achieved the expected competency levels satisfactorily. It was also found that 29.8% from Marassana, 39.1% from Thalathuoya, 51.7% from Galaha, and 53.4% students from Delthota have not successfully achieved the required competency levels. Furthermore, 28.1% students in Tamil medium and 58.0% students in Sinhala medium schools demonstrated poor performance. The results of the study revealed that current practices in Grade 5 Mathematics have to be re-evaluated to empower students for meaningful learning in Mathematics.

Keywords: Grade 5, Competency in Mathematics, Pathahewaheta education division

EFFECT OF STUDENT ENGAGEMENT AND TEACHERS' EMOTIONAL SUPPORT: A COMPARATIVE STUDY IN PRIMARY AND JUNIOR SECONDARY LEVEL SCHOOLS IN KINNIYA EDUCATION ZONE, EASTERN PROVINCE OF SRI LANKA

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This study explores the relationship between classroom interaction and students' emotional and behavioral engagement, with the aim of promoting optimal learning development and preventing school dropout. A survey was conducted involving 120 students from Grades 5, 8, and 9, with two classes selected from each grade across three schools in a specific education zone. Given the nested structure of data, multilevel analyses were employed, including descriptive statistics, confirmatory analysis, and multilevel structural equation modeling. The results revealed that students who perceived high-quality classroom interactions exhibited greater engagement in school. Notably, teachers' emotional support demonstrated the strongest association with engagement at both individual and classroom levels. Furthermore, the findings indicated that primary school students displayed higher levels of emotional engagement compared to lower secondary school students. Additionally, female students exhibited higher levels of behavioral engagement compared to their male counterparts. This study significantly contributes to the Sri Lankan Mathematics education community by emphasizing the significance of instructional support and student engagement to enhance overall performance in Mathematics.

Keywords: Behavioral and Emotional, Engagement, Teacher Support, Socioeconomic Status

**PEDAGOGY OF MATHEMATICS EDUCATION: A COMPARATIVE STUDY OF
EDUCATION SYSTEMS IN SRI LANKA AND FINLAND**

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This comparative study aimed to analyze and compare the school education systems of Sri Lanka and Finland. It employed a qualitative research method using thematic analysis, gathering, and analyzing secondary data from previous research and relevant sources. The study revealed significant differences between the two systems. The findings highlighted shortcomings in the Sri Lankan education system, characterized by a heavy emphasis on theory and exams. In contrast, the Finnish system prioritizes vocational training and practical assessments, de-emphasizing written exams. Moreover, Sri Lankan students face higher teaching and learning hours, leading to increased stress and reduced autonomy compared to their Finnish counterparts. The study also shed light on the misalignment between the Sri Lankan school curriculum and labor market demands, while the Finnish system showed a better connection between education and workforce needs. This research contributes greatly to the Mathematical community in multiple ways. It enhances cross-cultural understanding by examining two distinct education systems. It identifies effective practices in the mathematics classroom, serving as an inspiration for innovation and logical thinking. Additionally, the study provides policy recommendations to improve mathematics education in schools. In conclusion, this comparative study provides valuable insights into the education systems of Sri Lanka and Finland, highlighting the strengths and weaknesses of each.

Keywords: Comparison, Curricula, Finland Education System

ACTION RESEARCH TO DEVELOP STUDENTS' ABILITY TO USE BASIC MATHEMATICAL SKILLS IN PROBLEM-SOLVING

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Understanding basic mathematical skills is crucial for both work and everyday life. Currently, many students rely heavily on memorization when solving mathematical problems, leading to minimal comprehension of the mathematical concepts. Also, ineffective learning methods further impede their ability to grasp these concepts, directly affecting their problem-solving capabilities. The present study is action research aimed at providing hands-on experience in correctly applying basic mathematical operations to solve problems. Initially, three students who were struggling with problem-solving in the classroom (Grade 8) were identified based on their complaints. A test was conducted to assess their understanding of addition and subtraction involving positive and negative numbers. The results revealed a lack of comprehension regarding number lines. To address this issue, the first intervention involved introducing games that allowed students to grasp the concept of number lines and their role in solving problems. Active engagement and enthusiastic participation were observed, and the improved understanding was evident in their approaches to problem-solving. The second intervention involved introducing the abacus and employing various pedagogical strategies to teach students different methods of using it. Through this approach, students were given opportunities to solve problems of varying complexity. This intervention spanned approximately three months, during which two evaluations were conducted to monitor the progress. Throughout the process, it was observed that the student's ability to solve problems involving positive and negative numbers improved significantly. During the process, data were collected both quantitatively (test scores) and qualitatively (behavioral changes). Test scores were analyzed using MS Excel, while behavioral observations were analyzed through thematic analysis. The findings revealed that the three students demonstrated increased motivation to work cooperatively and enthusiastically during mathematical sessions. Furthermore, they showed a greater willingness to complete exercises, provide correct answers more often, and actively cooperate with the teacher and other students with an open mind.

Keywords: Abacus, Action research, Problem-solving, Thematic analysis

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Author Index

A		J	
Alwis, J.A.D.C.M.	12	Jayakody, G.N.	44
Amarasinghe, A.K.	28,48,53	Jayasinghe, J.D.T.U.K.	49
Arunmaran, M.	8	Jayasinghe, P.L.M.W.T.P.	6
Athapaththu, A.M.S.L.K.	36,42	Jayathilaka, L.R.M.K.R.	28
Athapattu, A.M.C.U.M.	36,37	Jayawardana, L.V.P.K.	51
B		K	
Bandara, B.M.S.	43	Kajan, N.	21
Bandara, R.M.V.V.	42	Kaluarachchi, K.N.	1
C		Kannan, K.	21
Chandralal, P.D.D.A.	39	Kapuhennayake, K.M.P.G.S.C.	31
Chandrasena, W.D.	47,48,49,53,54,55	Karunarathna, W.H.D.T.	3
Chandrasiri, A.M.P.	26	Korala, S.K.D.T.D.	9
Chathuranga, K.M.N.M.	37,56,57	M	
D		Mathanaranjan T.	24
Daundasekara, W.B.	2,17,32,35	Mohammad, M.A.M.	18,20,22,25
De Silva, K.H.C.	10	N	
De Silva, T.H.K.R.	26	Nawaratne, S.S.	25
Dewasurendra, M.T.M.	3,4,5,39,40,56,57	Nishantha, S.A.A.	22,23
Dhananjaya, K.D.E.	11	P	
Dheerasinghe, G.W.M.M.K.	11	Perera, A.A.I.	1,10,11,14,19,31,41
Dilshan, M.M.C.	31,41	Perera, H.N.	48,53
Dissanayake, D.M.S.D.	55	Perera, K.K.K.R.	12
Dissanayake, K.M.N.	3	Perera, M.K.	50,52
Dissanayake, R.B.N.	7	Perera, M.P.M.	45
E		Premachandra, P.K.	7
Ekanayake, E.M.D.B.	29	R	
Ekanayake, E.M.T.D.K.	13	Ranasinghe, P.G.R.S.	11,42
Ekanayake, E.M.U.S.B.	13,29	Randunu, M.W.S.	37
Eriyagama, L.S.K.	43	Rathnayake, D.R.N.N.	44
F		Rathnayake, P.	58
Fernandopulle, C.T.	20	Rodrigo, W.N.P.	27,30,32,34
G		Ruwan, S.M.T.	1
Gnanapragasam, S.R.	2	S	
Gunarathna, W.A	18,20,25	Sahabandu, C.W.	3,5
Gunarathne, R.	27	Sandaruwana, M.K.D.D.	15
H		Sandaruwini, B.A.P.N.	16
Hassan, S.M.	56	Selvarajah, P.	33
Herath, H.M.A.B.	47	Siyath, M.N.M.	57
Hisam, M.S.M.	32	Somathilake, L.W.	9,16
Hulangamuwa, R.R.W.G.B.M.K.B.	7	Subawickrama, H.D.A.W.	22
I		T	
Indunil, W.K.M.	1	Thalwaththe, S.T.R.D.	35
Infas, S.M.	30	Thilakarathna, D.	38
Ishara, A.A.D.H.	17	Thavarajah D.	21
		U	
		Udagedara, U.G.I.G.K.	22,23
		V	

Vidumali, M.W.M.L.	23
Viduranga, P.V.V.	15

W

Waidyaratne, H.R.H.S.	38
Wanamalee, K.R.	54
Warushavithana, D.M.A.	46
Weerabaddana, E.H.W.	19
Weerasinghe, G.H.S.N.	14
Weerasinghe, J.A.M.P.	18
Welgama, C.R.	34
Wickramarachchi, M.S.	6

Z

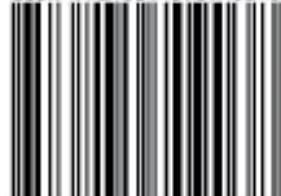
Zathiha, M.M.	40
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