

**POSTGRADUATE INSTITUTE OF SCIENCE
UNIVERSITY OF PERADENIYA
SRI LANKA**



PROCEEDINGS

Volume 2

**International Conference on Mathematics
and
Mathematics Education**

8th and 9th of October, 2021

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ICMME 2021

Message from the Director, Postgraduate Institute of Science



It brings me great pleasure to send this message in anticipation of the 2nd International Conference on Mathematics and Mathematics Education (ICMME) 2021, which will take place on October 8th and 9th, 2021 at the Postgraduate Institute of Science (PGIS), University of Peradeniya.

ICMME 2021 is co-organized by the PGIS and the Ministry of Education in Sri Lanka. Mathematics is a living discipline that attempts to comprehend patterns that pervade both the physical world and the mind within us. Even though mathematics is founded on rules, it is crucial for motivation that students go beyond rules to be able to express themselves in mathematical language. Many professions today require expert thinking and non-routine analytical abilities in order to identify and solve challenges. Mathematics education focuses on the development of an individual's analytical and problem-solving skills. As a result, a high-quality mathematics education will ensure that students learn abilities that are important not only in science and technology, but also in everyday life and the profession. Recognizing the importance of high-quality mathematics education, the Sri Lankan government has recently adopted a number of reforms. Although progress has been made, there is still space for improvement in certain critical components of the learning and teaching process.

The PGIS organizes and conducts many national and international conferences as well as numerous workshops and short courses. ICMME 2021 provides a platform for postgraduate research students and school teachers to highlight their findings, interact and exchange ideas. Therefore, I hope that this conference will bring together mathematics educators from across the country to provide such a forum as well as a platform for future collaborative activity. It is expected that the Conference would cover a wide range of areas in Mathematics and Mathematics education. All concerns are also expected to have distinct and supporting social aspects.

The Conference would not have been possible without the dedication and hard effort of our colleagues. We would like to extend our gratitude to the Conference Chair, Prof. W.B. Daundasekera and Dr. U.G.I.G.K. Udagedara, Secretary of ICMME 2021, for their planning and leadership in arranging this high-quality conference. We would like to thank the organizing committee members and reviewers for their significant assistance during the review process. We are also appreciative to all of the authors who entrusted their work to the Conference and would like to extend our congratulations to all of the presenters who will be sharing their findings at the Conference.

Prof. H.M.T.G.A. Pitawala

*Director/Postgraduate Institute of Science,
University of Peradeniya,
Peradeniya, Sri Lanka*

ICMME 2021

Message from the Chairperson



Dear Colleagues and Friends,

As the Chairperson I am delighted and honoured to bring this message to the International Conference on Mathematics and Mathematics Education 2021 organized by the Postgraduate Institute of Science (PGIS), University of Peradeniya, Sri Lanka with the collaboration of Ministry of Education, Sri Lanka. It is a pleasure and privilege to be part of this unique event. I take this opportunity to thank Vice Chancellor/University of Peradeniya, Dean/Faculty of Science, Director/PGIS, all the members of the organizing committee and subcommittees, and reviewers for their relentless dedication to bring this conference alive while facing the challenge of Covid – 19 pandemic.

This is an exciting day for us, because it is an attempt to enhance quality of research and education of Mathematics in Sri Lanka. I am privileged to be an academic in profession to use this as a vehicle to serve as the Chairperson of this conference to upgrade mathematics education in educational institutions. I always believe that improving and sustaining quality of education is paramount important to the development of any society and it lies mainly on innovative research conducted and practiced by researchers and educators. Also, I believe that the education is the greatest equalizer which enables a person to have a level playing field. Especially, mathematics gives us the critical ability to learn and think logically in any field of endeavor. The focal point of this timely event is to encourage and invite mathematicians and educators to share their findings and disseminate their knowledge in a common international forum.

I am pleased and very happy to inform you that we received a total of 91 research papers under two thematic areas: Mathematics and Mathematics Education from local and foreign authors. Each abstract was reviewed by at least two subject experts in the relevant field. After going through a strict and stringent review process, 80 papers were accepted for the Conference.

I sincerely hope that this event will bring the intellectuals together to make a significant impact upon the fields of Mathematics and Mathematics Education.

In conclusion, I wish you all good luck!

Prof. W. B. Daundasekera

*Chairperson
ICMME 2021*

ICMME 2021

Message from the Chairman/Board of Study in Mathematics



As the Chairman of the Board of Study in Mathematics, Postgraduate Institute of Science, University of Peradeniya, it's my utmost pleasure to send this message for the Second International Conference on Mathematics and Mathematics Education (ICMME) 2021 organized by the Postgraduate Institute of Science, University of Peradeniya in collaboration with the Ministry of Education.

The conference aims to share not only the knowledge and novel findings of our young researchers in Mathematics and Mathematics Education but also serves as a platform in sharing individual experiences in the respective fields.

Mathematics is aimed at developing the analytical thinking and problem-solving ability of a person which is lacking in the modern world. The objective of conferences of this nature is to fill up that void and to engage the mathematical community in disseminating knowledge.

The Board of Study in Mathematics at the Postgraduate Institute of Science has catered to produce postgraduate degrees and improve research in Mathematics. As a result, today our young researchers publish their research findings in both National and International Journals.

I would like to express my sincere gratitude to the Prof. H.M.T.G.A. Pitawala (Director/PGIS), Prof. W.B. Daundasekara (Chairperson of ICMME 2021), Members of the Organizing Committee, and the members of the Board of Studies in Mathematics and Science Education for their valuable support and guidance to make this great event successful.

Prof A.A.I. Perera

*Chairman/Board of Study in Mathematics
Postgraduate Institute of Science,
Sri Lanka*

ICMME 2021

Message from the Chairperson/Board of Study in Science Education



It's a pleasure to provide this message to mark the Second International Conference of Mathematics and Mathematics Education (ICMME) 2021 to be held on 8th and 9th October, 2021 with an opportune theme of 'Mathematics for a Better World'. There is no argument whatsoever to suggest that literacy in mathematics is a gateway to economic progress of countries, thus leaders of many western nations are trying best to enhance their performance indicators related to Mathematics and Science Education. As a result, policy makers place more focus on reassessment of the role of mathematics in the school curriculum. Not only that, acquiring a strong background in mathematics, students may not only bring about economic promise to the nation but also attain personal and social enrichment through improving skills such as imagination, creativity, analysis and synthesis. In reaching these goals, countries need a powerful force of well-equipped teachers to enact discipline-specific knowledge as well as subject-specific pedagogical knowledge. Thus, recognizing this national obligation, the Postgraduate Institute of Science, University of Peradeniya offers postgraduate degree programmes in Science Education in which Mathematics Education includes as a major discipline in the subject component. Moving one step forward, the PGIS held its first International Conference on Mathematics and Mathematics Education in 2019 with an encouraging participation from local and international contributors. This forum presents a suitable platform for educators to present their research findings in the field of Mathematics Education. The encouraging comments received from the participants from ICMME 2019 prompted us take a decision to conduct this event in a regular manner. This provides an excellent opportunity for educators as well as mathematicians to share their research findings while at the same time improving their knowledge and skills as well as build up effective collaborations. It is indeed a pleasure to be part of this energetic and enthusiastic team who organizes this event for the second time under challenging conditions due to the pandemic. Unfortunately, due to the prevailing conditions in the country, the Organizing Committee has taken a difficult decision to hold this ICMME 2021 on a virtual platform. Finally, I would like to take this opportunity to thank especially the younger members of the Organizing Committee who are actively and enthusiastically involve in order to make this event a reality. Their commitment is truly exemplary! Wish all participants a rewarding academic experience!

Prof. H.M.S.P. Madawala

Co-Editor-In-Chief – ICMME 2021

Chairperson/Board of Study in Science Education

Postgraduate Institute of Science

Sri Lanka

ICMME 2021

Message from the Secretary of the Ministry of Education, Sri Lanka



On behalf of the Ministry of Education, I am delighted to be invited as the Chief Guest for the auspicious occasion of the second installment of the International Conference on Mathematics and Mathematics Education (ICMME) 2021.

As the pioneer in legislation and the administrative hub in the Education sector in the country, Ministry of Education thrives in promoting and supporting in disseminating Education. No matter which field someone engages in, the prominence of Mathematics cannot be underwhelmed. ICMME 2021 is one of a kind conference in addressing not only Mathematics as an academic discipline but also giving emphasis to Mathematics Education in effective dissemination of knowledge and skills of Mathematics in the society.

Even at a challenging time as present due to the pandemic situation, the organizing committee has gone beyond their comfort zone in making this event a reality in virtual mode.

The inclusion and showcasing of the research findings is a very progressive step in the right direction in promoting Mathematics Education not only in the country but also in the global arena.

In conclusion, I wish to express my sincere appreciation to the organizing committee and all the personnel involved behind the scenes in making this event a success and I wish them all the very best in continuing events in this nature in future as well.

Prof. K. Kapila C. K. Perera

*The Secretary
Ministry of Education
Sri Lanka*

A Brief Biography of Professor Ratnasingham Shivaji



Professor Ratnasingham Shivaji joined the University of North Carolina at Greensboro (UNCG) as Head in July 2011 and served in this position until July 2019. Since January 2012, he is also serving as H. Barton Excellence Professor.

Prior to joining UNCG, he served for twenty-six years at Mississippi State University (MSU), where he was honored as a W.L. Giles Distinguished Professor. He received his Ph.D in Mathematics from Heriot-Watt University in Edinburgh, Scotland in 1981 and his B.S (first class honors) from the University of Peradeniya, Sri Lanka in 1977.

He is a Fellow of the American Mathematical Society. Shivaji's area of specialization is partial differential equations, and in particular, nonlinear elliptic boundary value problems. His research work has applications in combustion theory, chemical reactor theory, and population dynamics, and has been funded by the National Science Foundation and the Simon's Foundation. Currently, he is serving as the PI on an NSF Math Ecology grant.

To date, he has authored one hundred fifty-five research papers. He is a member of the Editorial Board of several mathematics journals. To date, he has directed one postdoctoral student, seventeen Ph.D. students (fourteen graduates, 3 current), fifteen M.S. graduates, and thirty undergraduate research students (twenty-eight former and 2 current).

The Abstract of the Keynote Speech of Professor Ratnasingham Shivaji

Uniqueness Results for Classes of Steady State Reaction Diffusion Equations

We will discuss uniqueness results for positive solutions to nonlinear boundary value problems of the form:

$$\begin{cases} -\Delta u = \lambda f(u); \Omega \\ u = 0; \partial\Omega \end{cases}$$

where Ω is a bounded region in \mathbb{R}^n ; $n \geq 1$, λ is a positive parameter, Δ is the Laplacian operator, and $f:[0,\infty) \rightarrow \mathbb{R}$ is a C^1 function.

A Brief Biography of Dr. Upali Mampitiya



Dr. Upali Mampitiya is a Senior Lecturer (Grade I) attached to the Department of Mathematics at the University of Kelaniya. He has the degrees of MSc and PhD in Mathematics from the University of Ottawa in Canada.

Currently he holds the following positions as well: Chairman-Academic Affairs Board/National Institute of Education, Member-Standing Committee on Accreditation and Quality Assurance (SCAQA)/Ministry of Higher Education, Member-Standing Committee on Quality Assurance/University Grants Commission, Member-Research and Development Committee/Ministry of Special Assignments.

He served as a member of the UNESCO Asia-Pacific Steering Committee on National Qualifications Frameworks during the period August 2015 - December 2016. UNESCO Bangkok office invited him to attend and make presentations at the Expert Meeting on Developing Regional Guidelines on National Qualifications Framework took place in July 2015 in Bangkok, Thailand and at the ERI-Net Annual Meeting: Case Studies on NQF for Asia-Pacific held in February 2016 in Tokyo, Japan.

He was the editor of “Updated Sri Lanka Qualifications Framework” published in December 2015 by the Ministry of Higher Education. In 2014, he assisted the National Education Commission as a member of the research team to conduct the “Study on Curriculum Development in General Education in Sri Lanka” a component of the proposals for a National Policy on General Education in Sri Lanka for the next decade. Also in the same year, he delivered the 24th Annual J E Jayasuriya Memorial Lecture on the topic “Mathematics Education – Past, Present and Future and he had the privilege of delivering the 30th Annual Dr. C.W.W. Kannangara Memorial Lecture - 2019, organized by the National Institute of Education (NIE). It was titled “Redefining Senior Secondary and Collegiate Curricular for Tomorrow’s World”.

The Abstract of the Keynote Speech of Dr. Upali Mampitiya

Developing 21st Century skills through Mathematics Education

This keynote address will first focus on the Sri Lanka Qualifications Framework (SLQF). The twelve categories of learning outcomes described by this framework are of national importance as they address the 21st century skills required by all qualification holders in the country, from senior secondary level upwards. As a practical application of the SLQF, the senior secondary mathematics curriculum will be looked at from the viewpoint of achieving those twelve categories of learning outcomes. Finally, the contribution expected from every teacher to help students achieve those learning outcomes will be discussed in terms of the learning-teaching methods and assessment techniques practiced in the classroom.

A Brief Biography of Professor Kuppalapalle Vajravelu



Kuppalapalle Vajravelu is a Professor of Mathematics and Professor of Mechanical, Materials and Aerospace Engineering at the University of Central Florida (UCF). He graduated from IIT, Khargapur with a PhD in 1979, and joined the Virginia Polytechnic Institute in the same year. Before his tenure at UCF, he also served as a visiting assistant professor at Eastern Carolina University.

He is an acclaimed researcher in the field of applied mathematics, specifically in Applied ODE and PDE, Fluid Mechanics, and Numerical Heat Transfer. He is an author of several influential books in the areas of Differential Equations, Nonlinear Mechanics and Homotopy Analysis. He has more than three hundred research papers (with a h-index of 54) in reputed journals and presented numerous talks and keynote lectures at international conferences.

As an administrator and an educator, Prof Vajravelu has a broad experience that spans for more than four decades. He was the associate chair of the Department of Mathematics (1999-2006) and was the Editor-in-Chief of the reputed open-access journal "Differential Equations in Nonlinear Mechanics" from 2005 to 2008. He is an editorial board member of several academic journals, including Journal of Applied Mathematics, Mathematical Problems in Engineering and Advances in Applied Research. He is also the founding Editor-in-Chief of the Journal of Differential Equations and Nonlinear Mechanics.

Prof. Vajravelu has been the recipient of numerous accolades throughout his illustrious career as an educator. He is a fellow of Andhra Pradesh Academy of Sciences, two-time awardee of undergraduate teaching award at UCF, recipient of Scholarship of Teaching and Learning Award, and five Teaching Incentive Programme awards.

**The Abstract of the Keynote Speech of Professor Kuppalapalle
Vajravelu**

A novel analytical method for systems of ordinary and partial differential equations

The optimal homotopy analysis method (OHAM) for the solutions of nonlinear differential equations arising in science and engineering will be presented. In order to obtain accurate approximate analytical solutions, multiple auxiliary linear operators will be considered to obtain better accuracy with relatively few terms. The convergence control parameters will be selected through the construction of an optimal control problem for the minimization of the accumulated residual errors. Also, open questions related to OHAM will be discussed.

Furthermore, a Directly Defining Inverse Mapping Method (MDDiM) to obtain solutions to nonlinear systems arising in combined free and forced convection flows will be presented. Solutions with errors ranging from five decimal accuracy to ten decimal accuracy will be obtained with relatively few terms. The idea is novel and is not limited to solving only ordinary differential equations. To demonstrate this, the MDDiM will be applied to solve systems of partial differential equations.

A Brief Biography of Dr. Boon Liang Chua



CHUA Boon Liang is an Assistant Professor in mathematics education at the National Institute of Education, Nanyang Technological University in Singapore. He holds a PhD in Mathematics Education from the Institute of Education, University College London, UK. His research interests cover pattern generalization, mathematical reasoning and justification, and task design.

Given his experience as a classroom teacher, head of department and teacher educator, he seeks to help mathematics teachers create a supportive learning environment that promotes understanding and inspire their students to appreciate the beauty and power of mathematics. With his belief that students' attitudes towards mathematics are shaped by their learning experiences, he hopes to share his passion of teaching mathematics with the teachers so that they make not only their teaching more interesting but also learning mathematics an exciting and enjoyable process for their students.

He feels honoured to have been awarded Excellence in Teaching by the National Institute of Education in 2009 and 2013. He developed teaching materials and conducted mathematics workshops for students and educators from many different countries, including Abu Dhabi, Brazil, Brunei, Cambodia, China, Guinea, Indonesia, Jamaica, Laos, Malaysia, Myanmar, the Philippines, Sri Lanka, Thailand, and Vietnam.

The Abstract of the Keynote Speech of Dr. Boon Liang Chua

**Enhancing Learning Experiences of Students through Mathematical Sense-making
and Reasoning**

Learning mathematics aims to equip learners with not just the necessary knowledge and skills but also the capacities to think logically and analytically, as well as to reason and communicate mathematically. The learning process is however fraught with difficulties for some learners. These learners may get frustrated when they are not able to make sense of the mathematics that they are learning in the classroom. Those who can make sense of the mathematics are likely to stay engaged in learning and enjoy it.

In this talk, I will share about how mathematics teachers can support learners and enhance their learning experiences through the provision of meaningful, engaging and stimulating learning activities and opportunities for them to articulate their reasoning clearly.

ICMME 2021

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**LINEARIZING INVERTIBLE POINT TRANSFORMATIONS AND
APPLICATION TO A NON-LINEAR THIRD-ORDER ORDINARY
DIFFERENTIAL EQUATION**

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We study the existence of linearizing invertible point transformations that map a non-linear third-order ordinary differential equation (ODE) to the Laguerre-Forsyth canonical form $u''' + a(t)u = 0$. We use the direct method which provides in terms of coefficients of the nonlinear third-order ODE the necessary and sufficient conditions for linearizability [Nail H. Ibragimov, Sergey V. Meleshko, *Linearization of third-order ordinary differential equations by point and contact transformations*, J. Math. Appl. 308(1)(2005) 266-289] and the means to construct the linearizing invertible point transformations for the underlying equation. The non-linear third-order ordinary differential equation which we consider here for application is a generalization of the equation studied in the work of Ibragimov and Meleshko. The authors obtained the linearizing invertible point transformations for the equation which gave rise to the canonical form with $a(t) = 1$. In the present work, we demonstrate the linearizability and show how one can construct the linearizing invertible point transformations for our non-linear problem by reducing it to the canonical form $u''' = 0$. These invertible point transformations have not been derived previously and are thus new.

Keywords: Linearization conditions, Non-linear third-order ordinary differential equations, Point transformation

REDUCING THE STANDARD MINUTE VALUE OF A SINGLE DESIGN USING AN ASSIGNMENT MODEL IN A REPUTED APPAREL MANUFACTURING COMPANY IN SRI LANKA

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Apparel is one of the largest industries that help to grow up the economy of Sri Lanka. Therefore, the apparel manufacturing companies are trying to acquire high-quality products by optimizing the production processes and quality improvements. The Standard Minute Value (SMV) of a single design (layout plan) plays a decisive role in the production process of apparel companies. The objective of this study is to formulate a mathematical model to reduce the SMV of a single design by assigning skilled workers to each operation. The study considered a single design of a production line in a reputed apparel manufacturing company in Sri Lanka and observed that the management of this company allocated the workers to each operation by using their expertise and basic statistics knowledge. Therefore, an assignment model has been implemented using the SMV values of each worker for a manufacturing process of a single design. The proposed model was solved using Lingo 17.0 version and it provides an SMV value of 6.4783 which is 0.7717 less than the current SMV value of the design. Moreover, the optimum SMV values and optimum workers for each operation have been calculated. This study concludes that the suggested assignment model can be applied to reduce the SMV value of any single design of a production line in apparel manufacturing companies.

Keywords: Assignment model, Layout plan, Standard minute value

PRIME LABELING OF THE GRAPH OBTAINED BY UNION OF n COPIES OF $K_{1,2}$ AND CL_n

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Graph theory is the branch of mathematics that is used to represent mathematical problems in discrete mathematics and combinatorics, as well as practical problems in biology, computer science, and information systems. Graph theory can be combined with other mathematical subjects such as number theory and linear programming to identify some special relationships with graphs. A graph $G = (V(G), E(G))$ with $|V(G)|$ vertices is said to have prime labeling if its vertices are labeled with distinct positive integers $1, 2, 3, \dots, |v|$ such that for each edge $uv \in E(G)$ the labels assigned to u and v are relatively prime, where $V(G)$ and $E(G)$ are vertex set and edge set of G , respectively. Therefore, the graph G has a prime labeling whenever any two adjacent vertices can be labeled using two relatively prime numbers. If we admit prime labeling for a graph, then that graph is called a prime graph. In this research, a prime labeling method is introduced for the simple undirected finite graph obtained by taking union of the star graph $K_{1,2}$ and the circular ladder graph CL_n . These new graphs are obtained by attaching $K_{1,2}$ at each external vertex of the circular ladder graph CL_n and it is proved that the subsequent graphs are prime when $n \geq 3$ and $n \not\equiv 1 \pmod{3}$. As a future endeavour, we wish to implement this result for a grid graph of the resultant graph and wish to develop a computer program to label the higher-order degree of these types of graphs.

Keywords: Cyclic ladder graph, Prime labeling

**WAREHOUSE SPACE OPTIMIZATION USING
A GOAL PROGRAMMING MODEL**

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Warehouse operations play a key role in manufacturing organizations, providing support to run production processes smoothly. As a part of warehouse operations in a supply chain, loading problems are a point that allows optimization to be carried out with significant cost implications and a considerable impact on the rest of the supply chain in manufacturing organizations. This study proposes a simple and effective goal programming (GP) model to optimally use warehouse storage space by efficient palletizing. The quantity of total pallets required per day is derived based on the available demand per day and other constraints related to warehousing operations in a multi-product manufacturing context. With the required moderate computational efforts and time, the Microsoft Excel Solver generated optimal solutions to the GP models analyzed in this study. This suggests that the proposed GP approach is an effective solution method to the problem of optimizing the number of pallets needed to fulfill the demand from each product type, maximizing warehouse space utilization. This approach can be used as a basis to solve optimization problems of this nature in a small-scaled warehouse.

Keywords: Goal programming, Palletization problem, Warehouse optimization

K-FRAMES IN QUATERNIONIC SETTING

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Quaternions are an extension of complex numbers from the two dimensional plane to four dimensional space and form a non-commutative division algebra. Due to the non-commutativity, there are two types of Hilbert spaces over quaternions, called right quaternionic Hilbert space ($V_{\mathbb{H}}^R$) and left quaternionic Hilbert space ($V_{\mathbb{H}}^L$). In this study the characterization of K - frames in left quaternionic Hilbert space is reported and the relation between Bessel sequence and K -frame also discussed. K - frames are more general than ordinary frames in sense that the lower frame bound only holds for the elements in the range of the K^* , where K is a bounded linear operator in $V_{\mathbb{H}}^L$ and K^* is its adjoint. Let $V_{\mathbb{H}}^L$ be a left quaternionic Hilbert space and $K \in \mathcal{B}(V_{\mathbb{H}}^L)$ a bounded linear operator. A sequence $\Phi = \{\varphi_k\}_{k \in I}$ is called a K - frame for $V_{\mathbb{H}}^L$, if there exist constants $m, M > 0$ such that $m\|K^*\varphi\|^2 \leq \sum_{k \in I} |\langle \varphi | \varphi_k \rangle|^2 \leq M\|\varphi\|^2$, for all $\varphi \in V_{\mathbb{H}}^L$. If $K = I_{V_{\mathbb{H}}^L}$, then Φ is an ordinary frame. The constants m and M are called lower and upper bounds of Φ , respectively. The K - frames can be characterized by a well-defined operator. That is, a sequence $\Phi = \{\varphi_k\}_{k \in I}$ is a K - frame if and only if $T_{\Phi}: \ell^2 \rightarrow R(T_{\Phi})$ defined by $\{q_k\}_{k \in I} \mapsto \sum_{k \in I} q_k \varphi_k$, is a well-defined operator and $R(K) \subseteq R(T_{\Phi})$. For a given K - frame $\{\varphi_k\}_{k \in I}$ a Bessel sequence $\{\psi_k\}_{k \in I} \subseteq V_{\mathbb{H}}^L$ is called a K - dual of $\{\varphi_k\}_{k \in I}$ if $K\varphi = \sum_{k \in I} \langle \varphi | \psi_k \rangle \varphi_k$, for all $\varphi \in V_{\mathbb{H}}^L$. We conclude that if $\{\varphi_k\}_{k \in I}$ and $\{\psi_k\}_{k \in I}$ are two Bessel sequences with $K\varphi = \sum_{k \in I} \langle \varphi | \psi_k \rangle \varphi_k$, for all $\varphi \in V_{\mathbb{H}}^L$, then $\{\varphi_k\}_{k \in I}$ and $\{\psi_k\}_{k \in I}$ are a K - frame and a K^* - frame respectively. K -frames provide a suitable mathematical frame work for applications in Physics and Engineering. Complex numbers are two dimensional while the quaternions are four dimensional, hence the structure of quaternionic Hilbert spaces are significantly different from their complex counterparts, and therefore, in the application point of view the theory developed in this note may provide useful generalization.

Keywords: K - frames, Quaternion, Quaternionic Hilbert space

HYPERGROUP-RINGS, THEIR DEFINITION AND SOME ELEMENTARY PROPERTIES

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Hyperstructures are algebraic structures which admit more than one output under the operation, which is called a hyperoperation. Hyperstructures are useful in describing physical phenomena such as particle interactions. In this research, we define hypergroup-ring $R(G)$ of a finite group G over a general hyperring R as a hyperstructure analogue of the group-ring. We show that the hypergroup-ring defined, is a general hyperring and we go on to prove some basic properties of hypergroup-rings, in particular, we look at certain class of hyperideals of hypergroup-rings and prove that the quotient structures that arise from these hyperideals are isomorphic to the hypergroup-rings over quotient hyperrings. Also, we show that if G and H are two finite groups, then $R(G \times H) \cong (R(G))(H)$. Finally, we identify an embedding from a hypergroup-ring $S(H)$ into another hypergroup-ring $R(G)$, provided S is a subhyperring of R and H is a subgroup of G .

Keywords: Group-ring, Hypergroup-ring, Hyperring, Hyperstructure

**A MATHEMATICAL MODEL TO INVESTIGATE THE EFFECTS OF
QUARANTINE AND ISOLATION ON THE SPREAD OF COVID-19**

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Maintaining disease control strategies such as social distancing, quarantining, isolating infectious people, and hygiene practices are very important in managing the spread of COVID-19. China has gained a relative success by practicing these control strategies effectively. Sri Lankan government could successfully control the first wave of the disease by implementing these control strategies effectively, but presently the second wave is spreading harmfully. Unfortunately, many countries have failed in the disease controlling process. This study proposes a compartmental mathematical model to investigate the effects of control strategies, quarantine, and isolation on the spread of COVID-19. In the formulation of the model, population subclasses susceptible (S), exposures (E), quarantine (Q), infectious (I), isolated (J), and recovered (R) are considered. The proposed model is a modification of the mathematical model proposed by Gummel et al. in 2004 for Severe acute respiratory syndrome (SARS) control strategies. In Gummel's model, the transfer behaviour from the quarantine class (Q) to the susceptible class (S) is not taken into account. Due to the fact that the quarantined people are released to the society, after their period of isolation, the transfer behaviour from Q to S is added to the proposed model. The disease dies-out parameter region is determined based on the basic reproduction number of the model. The effect of the lockdown on the controlling process of the disease is explained based on the results of the mathematical model. The model helps to identify better rates and applying stages of control strategies to control the disease, considering available health care capacity. The outcomes of the model show that the earlier the exposures are identified and quarantined, and the earlier the infectious people are identified and isolated, the better it helps in controlling COVID-19. It is observed that when all control strategies are applied simultaneously, the rate of the infected population curve flattens faster than when each control strategy is applied alone.

Keywords: COVID-19, Isolation, Quarantine, Reproduction number, SEQIJR models

MULTI-OBJECTIVE FUZZY OPTIMIZATION APPROACH TO SOLVE A KILN LOADING PROBLEM IN CERAMIC INDUSTRY

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One of the main challenges faced by many porcelain manufacturing companies is to maximize the kiln loading. In fact, kiln creates a bottleneck in the production process. Therefore, if the company can maximize the kiln loading, the production will be smoother resulting in higher profit. On the other hand, the number of kiln shelves (bats) used in the process is also to be considered in minimizing the production cost. The study aims to choose a competitive method to solve this problem by formulating a multi-objective linear programming (MOLP) model. Many decision support systems in manufacturing companies use the conventional linear programming approach to optimize a single objective function that maximizes the total number of units. But, this fails to address the problem properly when the production process has more decisions to be made. In this scenario, conflicting multiple objectives will come into the picture. The need to seek the optimal solution among the feasible solutions in properly posed problems is the rationale behind the theoretical approach and appropriate solution methods. Therefore, this problem is formulated as a MOLP model and solved using Goal Programming and Multiple Objective Linear Programming with Minimax principle techniques. Considering the uncertainty that prevails in the model, it was resolved under the fuzzy environment to seek a better near optimal solution. Hence, the problem was solved using two multi-objective fuzzy optimization techniques: Multi-Objective fuzzy linear programming with linear membership function and Multi-Objective fuzzy linear programming with S-Shaped membership function. Finally, a comparative analysis is performed on the optimal solutions obtained from all four techniques. The study concludes that for the kiln loading problem, among the techniques used, the most realistic near optimal solution is obtained by the Multi-Objective fuzzy linear programming technique with the S-Shaped membership function.

Keywords: Goal programming, Linear membership function, Multi-objective linear programming, Minimax principle, S-Shape membership function

SOLVING INTERVAL TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS USING ANT COLONY OPTIMIZATION ALGORITHM

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In the recent past, much attention has been focused on the Interval Transportation Problem (ITP) with mixed constraints, which is a special variant of Transportation Problem (TP), where the parameters of the model are interval-valued. In the literature we find many researchers have proposed various meta-heuristic techniques to solve the ITP. However, solving the aforementioned class of problems using a modified version of a biologically inspired algorithm known as Ant Colony Optimization (ACO) algorithm, based on Transition Rule and Pheromone Update Rule, has not been attempted. The foraging behavior of a colony of searching ants, and their ability to determine the shortest route between their nest and a food source, is analogous to ACO as an optimization mechanism. In this study, the ITP is converted to a crisp TP using the aforementioned modified ACO algorithm. By applying ITP with mixed constraints and ACO algorithm, the study calculates the optimal relationship between facilities, and the probabilistic approach proposed by this study to solve ITP is less complicated and more computationally efficient compared to the well-known meta-heuristic algorithms found in the literature.

Keywords: Ant colony optimization algorithm, Interval transportation problems, Lower and upper bounds and mixed constraints

MULTI-OBJECTIVE FUZZY-LINEAR PROGRAMMING APPROACH TO SOLVE BIN-PACKING PROBLEM

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Multi-objective linear programming problem (MOLPP) has important applications in many areas of engineering and management. In applications, one of the major challenges faced by researchers and decision makers (DM) is to estimate the parameters in MOLPP. Modelling and optimizing under a fuzzy environment are called fuzzy modelling and fuzzy optimization respectively. The Multi-objective fuzzy linear programming (MOFLP) approach is one of the most frequently used approaches in a fuzzy decision-making environment. Although it has been investigated and experimented for more than decades from various points of view, it is still worth to develop more advanced techniques in order to precisely fit the real-world problems within the framework of MOFLP. However, when formulating the MOLPP which closely describes and represents the real situation, various factors of the real system should be reflected by the description of the objective functions and the constraints. In this study, packing patterns are generated using the Modified Branch and Bound algorithm (MBBA) and the optimum packing arrangement is determined by solving an integer linear programming problem (ILPP) developed by Gilmore (model 1). A new ILPP (model 2) has been introduced to solve the Bin-packing problem (BPP) by minimizing the number of extra packing pieces at the optimal packing schedule. Under these circumstances, the MOFLP problem has been formulated to solve the BPP by minimizing the unused packing area while minimizing the number of extra packing pieces. A case study is considered to illustrate the MBBA and a far better solution was obtained for a packing schedule using MOFLPP than the solutions obtained from model 1 and model 2.

Keywords: Bin-packing problem, Matlab software, Microsoft Excel, Modified Branch and Bound algorithm, Multi-objective fuzzy linear programming

THE CHROMATIC POLYNOMIALS OF CIRCULAR LADDER GRAPHS

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The study of the chromatic polynomial associated with a special part in algebraic graph theory was initiated by George David Birkhoff. The chromatic polynomial counts the number of graph colourings as a function of the number of given k colours. The most popular theorem to obtain chromatic polynomials $P_G(k)$ of a graph G , is the deletion-contraction theorem. In our work, we consider circular ladder graphs (CL_n) which have $2n$ nodes and $3n$ edges and is equivalent to the generalized Petersen graph $P_{n,1}$. The chromatic polynomial $P_{CL_n}(k)$ can be constructed using the deletion-contraction method. Then it gives several intermediate graphs with a lot of calculations. Therefore, we propose a method to construct the chromatic polynomial of CL_n for $n > 3$ using the ladder graphs L_n and the circular ladder graphs CL_{n-1} and a specific graph Q_n . Each Q_n contains $n - 1$ number of C_4 graphs and a C_{n-1} graph. In our work, chromatic polynomials of circular ladder graphs have been derived with a smaller number of calculations and intermediate graphs. We proposed a result to find the chromatic polynomials of graphs and a conjecture is stated for the chromatic polynomial $P_{CL_n}(k)$ of CL_n ,

$$P_{CL_n}(k) = P_{L_n}(k) - 2P_{Q_n}(k) + P_{CL_{n-1}}(k).$$

Keywords: Chromatic polynomials, Circular ladder graphs, Deletion-Contraction method, Ladder graphs

**THE HYPERBOLIC TANH METHOD FOR SOLITARY WAVE SOLUTION OF
CAMASSA- HOLM- NONLINEAR SCHRÖDINGER EQUATION**

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The Camassa-Holm (CH) equation is a completely integrable nonlinear dispersive partial differential equation (PDE), which has attracted much interest due to its soliton solutions and its applications to shallow water waves. On the other hand, the nonlinear Schrodinger (NLS) equation is a completely integrable nonlinear dispersive PDE that has been fundamental towards shaping our understanding in a diverse array of systems. In this study, we consider the Camassa-Holm-nonlinear Schrodinger (CH-NLS) equation, which has the following form:

$$im_t + u_{xx} + 2\sigma m(|u|^2 - a^2|u_x|^2) = 0, \quad m = u - a^2u_{xx},$$

where $u(x, t)$ is a complex field, $\sigma = \pm 1$ pertains to focusing or defocusing non-linearity, a is a constant and subscripts denote partial derivatives. The CH-NLS equation is newly derived in the sense of deformation of hierarchies of integrable systems. The main objective of this study is to find the solitary wave solution of the CH-NLS equation using hyperbolic tanh method. This method is an efficient method for obtaining solitary wave solutions of many nonlinear evolution equations. As a result, we obtained the dark soliton solution for the above considered model. Futhermore, we discuss the geometric interpretation of the obtained exact solution.

Keywords: Camassa-Holm-Nonlinear Schrodinger (CH-NLS) equation, Homogeneous number of soliton solutions, Singularities

AN EXTENSION TO METHOD OF DIRECTLY DEFINING THE INVERSE MAPPING FOR PROPAGATION OF HARMONIC WAVES IN NONLINEAR GENERALIZED MAGNETO - THERMOELASTICITY WITH TWO RELAXATION TIMES UNDER INFLUENCE OF ROTATIONS

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There are several methods to solve non-linear differential equations. Perturbation Method, Homotopy Analysis Method (HAM), Optimal Homotopy Analysis Method (OHAM), and Method of Directly defining the inverse Mapping (MDDiM) are some of them. MDDiM is the most recent approximation method to solve non-linear ordinary differential equations introduced by Sujin Liao in 2016. Dewasurendra et al. (2018) extended this method to solve systems of coupled non-linear ordinary differential equations. Recently, we extended this method to solve single non-linear partial differential equations. In this study, we further extended this novel method to solve systems of coupled partial differential equations and applied it to study harmonic wave propagation in non-linear magneto-thermoelasticity media. The governing equation of this application contains two non-linear partial differential equations together with two initial and boundary conditions. The corresponding square residual errors of the third-order approximation solutions to the displacement, $u(x, t)$ and the temperature, $\theta(x, t)$ obtained by our extended MDDiM are respectively 6.6946×10^{-7} and 1.0892×10^{-5} whereas these errors for the real-world application by OHAM are 7.9675×10^{-7} and 2.1432×10^{-5} . Besides, though our extended MDDiM solves equations, the OHAM solves differential equations, taking more computational time than our method. Hence, the superior performance of our extended MDDiM is reported against OHAM in terms of the accuracy of the solutions and computational time.

Keywords: Harmonic wave propagation, Method of Directly defining the inverse Mapping, Optimal Homotopy Analysis method, Square residual error

E-SUPER VERTEX MAGIC LABELLING OF ODD ORDER BINARY TREE WITH AT LEAST TWO VERTICES OF DEGREE 2

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Graph labelling is an emerging area in the field of graph theory. Sedlacek (1963) defined the concept of magic labelling. There are several types of magic labelling defined on graphs such as edge magic labelling, vertex magic labelling, anti-magic labeling etc. In this study, we consider E -super vertex magic labeling. For a finite simple graph G , the set of vertices and edges will be denoted by $V(G)$ and $E(G)$ respectively. If G is a finite simple undirected graph with p vertices and q edges, then a vertex magic total labelling is a one-to-one map; λ from $V(G) \cup E(G)$ onto the set $\{1, 2, \dots, p + q\}$ with the property that for any vertex u ,

$$\lambda(u) + \sum_{v \in N(u)} \lambda(uv) = k.$$

Here, k is a constant and the set $N(u)$ denotes the vertices adjacent to vertex u . The labelling is called E -super vertex magic labelling, if the labels of the edge set under the vertex magic total labelling is the set $\{1, 2, 3, \dots, q\}$. A graph G is called E -super vertex magic, if it agrees with E -super vertex magic labelling. E -super vertex magic labelling has numerous applications in cryptography, networking and many other fields. There is an open problem; Every tree is an E -super vertex magic. We studied possibilities for the existence of E -super vertex magic labelling for some of the binary trees with odd order. In our work, we have proposed a result which can be used to find E -super vertex magic labeling for the binary trees. According to the proposed result, labellings of edges which adjacent to leaf vertex should be greater than or equal to $k - (p + q)$. Based on this lemma, it is possible to prove E -super vertex magic labelling does not exist for the perfect binary tree $T(2^{n+1} - 1, 2^{n+1} - 2)$ which has $2^{n+1} - 1$ vertices, and $2^{n+1} - 2$ edges where n is the height of the tree, and E -super vertex magic labelling exists for odd order binary trees with at least two vertices of degree 2.

Keywords: E -super vertex magic labelling, Super vertex magic labelling, Tree graph

ESTIMATION OF CAPACITOR FACTOR BASED ON A NEW POWER CURVE APPROXIMATION METHOD

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Modeling of a power curve data is an essential requirement in construction of many indicators which measure the performance of wind turbines. In this study, a novel approach for a interpolation model has been introduced to approximate the power curve data provided by a manufacturer. In the formulation of the new interpolation function, it has been verified that the sufficient conditions for the existence of a point of inflection in the specified range of velocity. Three versions of the interpolation formula, specified by the order of the polynomial, were applied to approximate the power curve data of 1.5 MW and 2.0 MW wind turbines. In order to compare the accuracy of the new interpolation model, both sets of power curve data were also approximated to the conventional polynomial form model. Then the capacitor factors of the wind turbines which were evaluated using the generic equations of power curve data and the wind speed distribution were obtained with respect to a wind speed dataset that had been recorded at Kokkilai of the North-eastern coast, Sri Lanka from February 2015 to February 2016. In this work, two missing blocks of original wind speed data were imputed using a simple regression-based approach with proper validation and the processed wind speed data were approximated to a Weibull distribution. The third order interpolation formula of the proposed method has given the most successful generic equations that approximate the power datasets of each wind turbine. Therefore, the capacitor factor values which were calculated based on the new third order interpolation formula are more effective than the values of the conventional method. According to the capacitor factor values, installation of the wind turbine of 2 MW capacity has shown economically feasible and efficient in comparison with the installation of the wind turbine of 1.5 MW.

Keywords: Capacitor factor, Data imputation, Interpolation, Point of inflection, Weibull distribution

SEMI-ANALYTIC SOLUTIONS FOR THE TRANSITIONAL KORTEWEG-DE VRIES EQUATION VIA SIMILARITY SOLUTIONS

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The Transitional Korteweg-de Vries (t-KdV) equation is a third-order non-linear partial differential equation that arises in the study of long solitary waves in lakes and estuaries. This study is mainly concerned with the similarity solution for the t-KdV equation; this is the first time one has used a similarity transformation to study the t-KdV equation. We first transformed the t-KdV into a third-order non-linear ordinary differential equation using a similarity transformation to obtain the similarity solution. We considered two special cases by comparing the coefficient of each term of the transitional Korteweg-de Vries equation. Subsequently, we solved both cases analytically via the Optimal Homotopy Analysis Method (OHAM). To validate the similarity solutions, comparisons were made with solutions obtained by the Method of Directly Defining the inverse Mapping (MDDiM). Also, the OHAM solutions for the several sets of parameters were obtained and presented.

Keywords: Analytical approximate solutions, Homotopy Analysis method, Method of Directly defining inverse mapping, Series solutions, Transitional Korteweg-de equation

AN ALTERNATIVE METHOD OF CONSTRUCTING SKEW-HADAMARD MATRICES OF ORDER 2^n

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The Hadamard matrix, named by the French mathematician Jacques Hadamard, is an $n \times n$ square matrix H whose entries are ± 1 satisfying $HH^T = nI_n$, where H^T is the transpose of H and I_n is the identity matrix. It is well-known that the order of a Hadamard matrix is 1, 2, or a multiple of 4. Skew-Hadamard matrices are of special interest among those. If H is a Hadamard matrix and $H + H^T = 2I_n$, then H is called a Skew-Hadamard matrix. These are used to construct several combinatorial objects. Though the constructions of most Hadamard matrices are known, not all of those give skew-Hadamard matrices. There are various methods to construct skew-Hadamard matrices such as Paley construction, Williamson construction, Goethals-Seidel construction, Wallis-Whiteman construction, and many more. In this research, an alternative method was proposed to construct skew-Hadamard matrices. First, 2×2 skew-Hadamard matrices with entries ± 1 were considered. Using the skew-Hadamard matrix $H_2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and H_2^T , we can construct the skew-Hadamard matrix H_4 of order 4. Then skew-Hadamard matrix H_8 of order 8 can be constructed using the skew-Hadamard matrices H_4 and H_4^T . This doubling construction method can be used to construct skew-Hadamard matrices of orders 2^n ($n \geq 2$) and the general form of the proposed method is given below:

$$H_{2^n} = \begin{bmatrix} H_{2^{n-1}} & H_{2^{n-1}} \\ -(H_{2^{n-1}})^T & (H_{2^{n-1}})^T \end{bmatrix}.$$

In future, we plan to implement a computer program to construct large skew-Hadamard matrices of order 2^n .

Keyword: Doubling construction method, Hadamard matrices, Skew-Hadamard matrices

FINDING AN OPTIMUM ASSIGNMENT FROM FUEL DISTRIBUTION CENTRES TO FILLING STATIONS IN THE CENTRAL PROVINCE, SRI LANKA

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Transportation and delivery are important decision factors in minimizing production cost in industries. Optimum assignments play a major role in minimizing transportation and delivery cost. Solving an Assignment Model has been a demanding subject since the inception of the field of Operations Research which dated back to the mid-1940s. Many researchers have made various attempts to solve the model using different approaches. In 1955, Harold Kuhn was able to develop an algorithm known as the Hungarian Algorithm (Wate-Mizuno, 2014). Assignment problem has many applications in industries to control and optimize different activities such as scheduling, transportation, etc. In this study, our focus was to search for an optimal assignment from fuel depots to filling stations located in the Central Province of Sri Lanka. At present, this assignment is prepared manually by the Management Department. This is performed with an irregular manner by the administrative staff. After scrutinizing the assignment schedule, several drawbacks were identified such as time spent on the activity and transportation cost. Also, the current practice does not fulfill certain requirements imposed by the Ceylon Petroleum Corporation (CPC). This stimulated us to conduct the study to overcome these prevailing drawbacks. The study was conducted on data collected in the year 2019 from the three fuel depots located in the Central Province at Peradeniya, Kotagala, and Kurunagala which are operated by the CPC. These depots are responsible for meeting the demands of the filling stations. An assignment model was developed to optimize the transportation cost from depots to filling stations while satisfying the demands of the stations and other resource limitations. The solution was compared with the existing schedule, which revealed that if the proposed solution is implemented, the CPC can save up to 0.5 million LKR annually. Therefore, the outcome obtained by solving the model is very promising and encouraging. Moreover, the CPC does not have to make any additional effort to implement the proposed method but save a significant amount of money by reducing working hours spent on scheduling the delivery as well as saving money on transportation and the other expenditure associated with it.

Keywords: Assignment model, Hungarian algorithm, Transportation cost

COCYCLIC MATRICES AND GENERALIZED HADAMARD MATRICES

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Over the years, generalized Hadamard matrices have been an active area of research. Let C be a finite abelian group of order w . A $v \times v$ matrix $M = [m_{ij}]$ with entries from C with $w|v$ is a generalized Hadamard matrix denoted by $GH(w, v/w)$ over C if, for all $i \neq j$, the sequence of quotients $m_{ij}m_{jk}^{-1}, 1 \leq k \leq v$, contains each element of C exactly v/w times. This empirical study attempts to construct generalized Hadamard matrices using cocyclic matrices which have the form $\psi = [\psi(g, h)]g, h \in G$, where G is a finite group and $\psi: G \times G \rightarrow C$ is a cocycle; that is,

$$\psi(g, h)\psi(gh, k) = \psi(g, hk)\psi(h, k), \forall g, h, k \in G.$$

Here, we restrict it to the normalized cocycles, for which $\psi(1, 1) = 1$. Moreover, ψ is said to be orthogonal if, for each $1 \neq h \in G, \sum_{g \in G} \psi(h, g) = v/w \sum_{a \in C} a$ in $Z[C]$. A cocyclic matrix $M\psi$ is $GH(w, v/w)$, if the cocycle ψ is orthogonal. Any cocycle can be expressed as a Hadamard product on inflation cocycle ψI , a coboundary $\partial\psi B$ and a transgression cocycle ψT . The main objective of this study was to provide a construction of transgression cocycle over \mathbb{Z}_2^n for $n \in \mathbb{N}$ and the study initiated with the transgression cocycle over \mathbb{Z}_2 . It has been investigated that the transgression cocycle over \mathbb{Z}_2^n is a two component mixture,

$$M\psi T (\mathbb{Z}_2^n) = \begin{bmatrix} M\psi T (\mathbb{Z}_2^{n-1}) & M\psi T (\mathbb{Z}_2^{n-1}) \\ M\psi T (\mathbb{Z}_2^{n-1}) \circ K_{2^{n-1}} & M\psi T (\mathbb{Z}_2^{n-1}) \circ K_{2^{n-1}} \end{bmatrix}, \text{ where } M\psi T (\mathbb{Z}_2^{n-1}) \text{ is the}$$

transgression cocycle over \mathbb{Z}_2^{n-1} and $K_{2^{n-1}}$ is the matrix obtained from the elements in \mathbb{Z}_2^{n-1} with a pattern. The method was tested for $n = 1, 2, 3$, and 4. The proposed method was extended to obtain a generalized Hadamard matrix over \mathbb{Z}_2^4 . It is a cocyclic matrix developed over \mathbb{Z}_2^4 and is a Hadamard product of

$$\mathcal{M}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, A, B, C, D) \circ \mathcal{M}(1, 1, 1, 1, 1, G, H, I, S, T, U, V, W, X, Y, Z) \circ \mathcal{M}(1, 1, 1, 1, 1, 1, 1, 1, 1, K, L, M, P, Q, R), \text{ where } A^2 = B^2 = C^2 = D^2 = \dots = R^2 = 1.$$

Keywords: Coboundary, Cocycles, Generalized Hadamard matrices, Inflation, Transgression

PALEY GRAPHS AND FACTORIZATIONS

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Graphs are mathematical structures represented by a set of vertices connected by edges. A distinction is made between undirected graphs where edges link two vertices symmetrically, and directed graphs where edges link two vertices asymmetrically. There are special types of graphs such as strongly regular graphs, bipartite graphs, Paley graphs, conference graphs and many more. Paley graphs are special types of graphs. They are dense undirected graphs constructed from the members of a suitable finite field for $q = p^n \equiv 1 \pmod{4}$, where p is a prime number and n is a positive integer. In Paley graphs, two vertices are adjacent if and only if their difference is a quadratic residue. When q is a prime power, exactly half of the non-zero elements of the *Galois field of order q* ($GF(q)$) are quadratic residues and exactly half of them are quadratic non-residues. Quadratic residues gives well-known Paley graphs which are regular of degree $(q - 1)/2$. In our work, we used quadratic non-residues as well and identified those graphs as regular of degree $(q - 1)/2$. These graphs are complements of graphs obtained from quadratic residues. The union of these two graphs gives vertex degree $q - 1$ in which all the edges are distinct and the total degree is $q(q - 1)$, and by the Handshaking Lemma, the number of edges is $q(q - 1)/2$. Therefore, this represents a complete graph K_q and the two graphs are $(q - 1)/2$ factors of K_q , and their union is a $(q - 1)/2$ -factorization of K_q . When q is a prime power and $q \equiv 1 \pmod{4}$, our result gives an alternative construction of factors of K_q . We are planning to generalize our work to construct factors of other special types of graphs using the theory of finite fields.

Keyword: Handshaking lemma, Paley graph, Quadratic non-residue, Quadratic residue, Regular graph

ON PRIME LABELING OF SNAKE GRAPHS

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A snake graph $C_{k,q}^m$ is the fusion of m number of k -cycles C_k , such that, for $2 \leq i \leq m$, a shared vertex called the *vertebrae*, denoted by v_i , results from the fusion, where a minimal path of length q joins v_{i-1} and v_i . In the present work, we focus on a few questions on prime labeling of snake graphs as stated in Bigham et al. Firstly, we consider the prime labeling of snake graphs when each cycle of snake does not have the same k or q values and secondly, the maximum m for which $C_{k,2}^m$ when k is odd and ≥ 3 . To find the prime labeling for snake graphs for different k values, we label the graph using the modified snake labeling when $q = 2$ by considering all the possibilities. To find the maximum value of m , we implement a computer program using python language from which we also obtain the smallest prime factor of $k - 2$ for all odd numbers up to any given number. Moreover, we are able to easily find all odd numbers between 3 and n (for any positive integer n), prime factors and the smallest prime factor of each odd number, and finally the $k - 2$ value without any errors. Considering the above results, we obtain a general pattern for the maximum m value. For the first question raised by Bigham et al., to find the prime labeling for snake graphs for different k values, we label the snake graph using the modified snake labeling when $q = 2$. As a result, we get 21 as a vertebra when labeling the graph using the modified snake prime labeling for $q = 2$, but have only 15 vertices. Also, in the above labeling, there are some integers missing (e.g., 3,7,8,14,15, and 16). When we consider all the possibilities of ordering, the snake graph cycles of $C_{4,2}^1$ and $C_{5,2}^1$ cannot be ordered as the 4th and 5th respectively, otherwise we ended up getting the same value for vertebrae. Also, when we change the order of the cycles, we again end up with getting the same value twice for spine and belly vertices. As a new approach to get a proper prime labeling for the snake graph, we find another prime labeling by increasing the cycles according to the results we obtain in the second question. Furthermore, by using the results of the computer program, we are able to obtain some general patterns for the m value.

Keywords: Cycles, Graphs, Prime labeling, Snake graphs

ON COMPOSITION OF FUNCTIONS DEFINING DISTINGUISHED VARIETIES

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This study aims to investigate the composition of distinguished varieties in terms of the functions defining them. The distinguished variety V is a non-empty subset of \mathbb{D}^2 such that V is the zero set of a polynomial $p(z, w) \in \mathbb{C}[z, w]$ in \mathbb{D}^2 and the extension of V to \mathbb{C}^2 contained in $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where \mathbb{D} is the open unit disc, \mathbb{T} is the unit circle and \mathbb{E} is the exterior of the closed unit disc in \mathbb{C} . We call such polynomials, $p(z, w)$ polynomials defining distinguished varieties. Finite Blaschke products generate distinguished varieties in such a way that, given a finite Blaschke product $b(z) = \beta \prod_{k=1}^n \left(\frac{z-a_k}{1-\bar{a}_k z} \right)^{m_k}$, where $\beta \in \mathbb{T}$, $a_k \in \mathbb{D}$ for $k = 1, 2, 3, \dots, n$, and m_k are the multiplicity of zero a_k , the numerator of $w^m - b(z)$ defines a distinguished variety. An automorphism on the unit disk has the form $f(z) = \frac{az+b}{\bar{b}z+\bar{a}}$ for $a, b \in \mathbb{C}$ with $|a|^2 - |b|^2 = 1$ or alternatively $f(z) = e^{i\theta} \frac{z-c}{1-\bar{c}z}$, where $c \in \mathbb{D}$ and $\theta \in (0, 2\pi]$. An automorphism on the unit bidisc is a pair (f_1, f_2) such that each f_1 and f_2 is an automorphism on the unit disc. In this study, we prove that composition of a polynomial generated by a Blaschke product that defines a distinguished variety with an automorphism on the bidisc, we get back a polynomial that defines a distinguished variety.

Keywords: Automorphisms, Blachke products, Distinguished varieties, Inner toral polynomials

ON THE SUM OF INNER TORAL POLYNOMIALS GENERATED BY
BLASCHKE PRODUCTS

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A polynomial $p \in \mathbb{C}[z, w]$ is said to have bidegree (n, m) if p has degree n in z and m in w . Given a polynomial $p \in \mathbb{C}[z, w]$ of bidegree (n, m) , the reflection of p at the bidegree (n, m) is defined to be the polynomial \tilde{p} given by $\tilde{p}(z, w) = z^n w^m \overline{p\left(\frac{1}{z}, \frac{1}{w}\right)}$. Further, p is called essentially \mathbb{T}^2 -symmetric if $p(z, w) = c\tilde{p}(z, w)$ for some unimodular constant c , and \mathbb{T}^2 -symmetric if $p(z, w) = \tilde{p}(z, w)$. We call this constant c , the reflective coefficient of p . A polynomial $p \in \mathbb{C}[z, w]$ is called inner toral if its zero set is a subset of $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where \mathbb{D} is the open unit disk, \mathbb{T} is the unit circle, and \mathbb{E} is the exterior of the closed unit disk. It was proven that inner toral polynomials are essentially \mathbb{T}^2 -symmetric. In this work, we focused on inner toral polynomials generated by Blaschke products. Infact, given a finite Blaschke product, $B(z) = \zeta \prod_{k=1}^{n'} \left(\frac{z-a_k}{1-\bar{a}_k z} \right)^{m_k}$, where $\zeta \in \mathbb{T}$ and $a_k \in \mathbb{D}$. The numerator of $w^m - B(z)$ is an inner toral polynomial of bidegree (n, m) , where $n = \sum_{k=1}^{n'} m_k$. In this study we present two results on inner toral polynomials generated by finite Blaschke products: (1) the reflective coefficient of an inner toral polynomial generated by finite Blaschke product $B(z)$ is $-\bar{\zeta}$, and (2) sum of the inner toral polynomials generated by Blaschke products of the form $B_k(z) = \zeta_k \frac{z-a_k}{1-\bar{a}_k z}$ for $k = 1, 2$ is inner toral if and only if $\zeta_1 = \zeta_2$.

Keywords: Blaschke product, Distinguished varieties, Inner toral polynomial, Reflective coefficient

PLUMERIA ALBA GRAPH AND ITS PRIME LABELING

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This work investigates the prime labeling of new graphs construct using complete tripartite graphs and their modification. Let $G = (V(G), E(G))$ be a graph with a set of vertices and a set of edges denoted by $V(G)$ and $E(G)$, respectively. Let graph G has $|V(G)| = n$. Then it admits a *prime labeling* if there exists a bijection map $f : V(G) \rightarrow \{1,2,3, \dots, n\}$ such that for each edge $e = uv$ in $E(G)$. The greatest common divisor (gcd) of adjacent vertices $f(u)$ and $f(v)$ is equal to 1 and $f(u)$ and $f(v)$ are called relatively prime. A Star graph is a particular type of complete bipartite graph, and the complete tripartite graph is a simple tripartite graph in which each vertex in one partite set is adjacent to all the vertices in the other two partite sets. In our present work, we introduced a new type of graph that has obtained by replacing every edge of the star graph $K_{1,n}$ by $K_{1,m,1}$ and then joining the vertices in each third partite sets of $K_{1,m,1}$ by an edge. The resulting graph is called the *Plumeria Alba* graph because of its shape of the flower, and it is denoted by $A_{m,n}$. The $A_{m,n}$ graph has $n(m + 1) + 1$ number of vertices. Finally, we proved that the resultant graph is a prime graph when $n \not\equiv 1(\text{mod } 3)$ for $m = 3$ and $n \not\equiv 1(\text{mod } 6)$ for $m = 5$, where $n > 1$.

Keywords: Complete tripartite graphs, *Plumeria Alba* graph, Prime graphs, Prime labeling

A NOVEL MODEL TO SEEK OPTIMAL EMPLOYEE TRANSPORTATION SYSTEM IN APPAREL INDUSTRY USING VEHICLE ROUTING METHOD

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One of the demanding and highly regarded problems in logistics is the Vehicle Routing Problem (VRP). The objective of the original model formulation of VRP is to find an optimal route for a delivery truck leaving from a single depot and returning to the same depot after visiting the assigned delivery centres. Over the years, the problem formulation of VRP has been heavily modified, and the objective of VRP varies depending on the application. The goal of this study is to develop an optimization model based on the original VRP model formulation for employee transportation. We specifically focus on an employee transportation problem related to MAS Intimates located in Pallekele by developing a mathematical model to find the shortest route to pick employees from different locations with the minimum number of vehicles. Twenty-one nearby locations were identified, and the number of employees waiting at each location varies between 3 and 12. It is assumed that a vehicle can accommodate approximately half of its capacity to maintain the social distance of passengers due to Coronavirus pandemic situation. Excel is used with OpenSolver (Linear Programming Simplex Algorithm) to implement and determine the optimal solution to the proposed model. Three different vehicle types with respect to capacity: van, coach, and bus, were considered and for the given vehicle type the capacity of the vehicle is considered to be constant in the model. The performance of the proposed model is compared against the traditional model formulation. Specifically, the proposed model provides a promising solution when fixed and variable costs of hired vehicles are taken into consideration.

Keywords: Apparel industry, Employee transportation, Vehicle routing problem

A CRITICAL ANALYSIS ON INTEGRATED INVENTORY SUPPLY CHAIN

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The integrated inventory supply chain (IISC) is a usual practice in the global business sectors and gives financial benefits to both the vendor and the buyer. Many researchers concern about the coordination of these two parties in the inventory system. This study contributes a systematic literature review in an IISC management system. The purpose of this study is not only to give a state-of-the-art of review in the IISC but also to provide the cooperative key themes of disseminations from 2011 to 2020. The number of published articles in the field of the IISC has increased significantly in recent years. So far, no attempts have been made to analyze these published articles utilizing bibliometric and network analysis software. Thus, this study utilizes VOSviewer software to perform a quantitative literature review and gives a low dimensional perception, where articles are situated in such a manner that the distance between any pair of items mirrors their similitude as precisely as could really be expected. In this work, 327 journal articles were selected for analysis and review based on keywords searched in the Scopus database last 10 years. The bibliometric information of selected papers was then analyzed using VOSviewer. This bibliometric analysis carried out co-word and co-citation analysis to quantitatively extract and analyze the contents of these articles. The analysis exposes that there is a gap amid the industrial and academic wise that desires to be spanned; further research is needed in the field of IISC management and needs to develop models for multi-vendor and multi-buyer criteria with consideration of realistic factors. Therefore, due to the increasing importance of integrated inventory management in today's global business perspective and sustainable survival and success of organizations, more enhancement studies are needed.

Keywords: Integrated inventory, Supply chain, VOSviewer

**OBTAINING BOUNDS TO A MULTI-OBJECTIVE TRANSPORTATION
PROBLEM WITH VARYING DEMANDS AND SUPPLIES**

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This study introduces a mathematical model for a transportation problem consisting of multi objectives with varying demands and supplies within intervals. According to the literature, there is only a few research has been conducted related to this problem. Most of the researchers have given an appreciable attention to minimize the transportation problem of single objective with varying demands and supplies, and multi objectives with fixed supply and demand. Apparently, in real life situations, most of the problems are a combination of these two types. Therefore, it is more important to determine the lower and upper bounds for the multi objectives with the ideal boundary solutions to make an acceptable decision. Here, we proposed a solution method with re-writing a bi-level mathematical model and using the fuzzy approach to obtain the bounds for a multi objective transportation problem with varying demands and supplies within intervals. The outcomes of the randomly generated problems demonstrate the significance of the proposed method.

Keywords: Multi-objective, Transportation problem, Varying demands and supplies within intervals

SCHEDULING A TELEVISION ADVERTISING CAMPAIGN USING A MULTI-OBJECTIVE NON-LINEAR PROGRAMMING MODEL

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In the world of business, advertising plays a major role in promoting products to the customers. Among various types of advertisements, television (TV) advertisements proven to have a greater potential in reaching a higher number of viewers. The scheduling of an advertisement in a given TV station to reach the maximum exposure is infact a daunting task. Therefore, it must be planned carefully considering the factors: best time-slots, TV programmes and budget allocation, to get the maximum exposure. In literature, it can be found that scheduling has been formulated as a deterministic as well as probabilistic models. In most current research, a goal programming approach was used to design a multi-objective TV advertising schedule. In our study, the advertising schedule is formulated as a multi-objective integer non-linear programming model, where the impact on the viewers on the advertisement follows according to Fechner's Law which assumes that impact on viewers from the advertisements is proportional to the logarithmic relation. The model was tested on three popular local TV stations, ITN, TV Derana and Hiru TV, where the TV programmes were telecasted in the evening session which was divided into eight-time slots with each thirty-minute period. It was assumed that the advertisement is repeated at the maximum three times during each time slot. Data which were collected from a reliable organization, consist of several piece of information such as the number of male and female viewers watching TV at different time slots, TV rate cards for collecting advertising cost. The model was solved using the Lingo software with the help of MS Excel for data anlalysis. The solution indicates the number of times the advertisement is repeated in each time slot to maximize the number of male and female viewers while satisfying the advertisement requirements including the budget.

Keywords: Advertising schedule, Fechner's Law, Lingo

A LINEAR MODEL TO IMPROVE THE PADDY HARVEST IN SRI LANKA: A COMPARATIVE STUDY USING RIDGE AND LASSO REGRESSION

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The central feature of Sri Lanka cuisine is boiled or steamed rice. The current paddy harvest of Sri Lanka only satisfies around 95% of the domestic requirement, and the remaining is imported. It is understood that the demand for milled rice is proportional to the population grows, and the current population growth rate is approximately 0.40%. In February 2020, the government took a policy decision not to import milled rice. Thus, it is in need to take the necessary actions to improve local production to meet the demand. It believes that paddy harvest depend on many human and naturally controlled matters. In this study, we have focused on the paddy production of each district and the effect of Temperature (^oC), Rainfall (mm), Humidity (%), UV index, Urea (kg/ha), Muriate of Potash (MOP) (kg/ha), and Triple Super Phosphate (TSP) (kg/ha) on paddy harvest. The goal of our study is to develop a linear model with high accuracy to predict the paddy harvest concerning important human and naturally controlled matters. The dataset consists of paddy harvest data from 2015 Maha to 2019 Maha along with eight predictor variables. The proportion of variance in paddy harvest that is predictable from the independent variables is 83% for the fitted ordinary least square linear regression model. Based on the *p*-Values, temperature, humidity, and TSP are the most statistically significant variables. Indication of multicollinearity among variables is minimum and fitted multiple linear regression models are accurate. According to the lasso regression model, urea, humidity, and TSP are the most important variables and UV index, humidity, and temperature are the most important variables in the ridge regression model. The accuracy of ridge and lasso regression models are 88% and 85% respectively. Comparing all three linear regression models, humidity is a dominant variable for predicting paddy harvest. Based on the model summaries, ridge regression provides a usable and accurate linear regression model for collected data. All the statistical analyses in the study were carried out using RStudio.

Keywords: Lasso regression, Paddy harvest, Rice, Ridge regression

A NOVEL CRYPTOSYSTEM USING A COMBINATION OF THE DIFFIE HELLMAN KEY EXCHANGE AND THE RSA ALGORITHM

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The word “cryptography” has Greek origin; *kryptos* means “hidden” while *graphein* means “to write”; in essence, “cryptography” is the art of writing codes for secure communication which plays an imperative role in the day-to-day life. The first formulation of a public key cryptosystem was introduced around 1970 by Wilfred Diffie and Martin Hellman; the Diffie Hellman key exchange, which provided a new direction to cryptography. Later, in 1977, Ron Rivest, Adi Shamir, and Leonard Adleman introduced the most popular RSA algorithm which has a higher security even today. In this study, we propose a new public key cryptosystem combining both the RSA algorithm and the Diffie Hellman key exchange. Our algorithm can be categorized under the three main aspects; key generation, encryption, and decryption. The key generation is developed with the use of the Diffie Hellman key exchange and binary addition while the encryption and decryption processes are based on techniques of the RSA algorithm. Here, the plaintext was converted into bits and the shared key was added via binary addition before the RSA encryption providing an authentication. As we combine both algorithms, the security of the proposed algorithm is achieved by the *discrete logarithm problem* from Diffie Hellman key exchange and *factorization assumption* from the RSA algorithm, both of which are considered as advantages. The difficulty of factoring large integers is known as the *factoring assumption* and no efficient algorithm exists to solve this problem in polynomial time with large enough keys of size 1024 bits or more, adding higher security level to the proposed algorithm. In addition, the *discrete logarithm problem* or the difficulty of computing logarithm over a given prime under a given modulo ensures the security of the shared key. Furthermore, the proposed scheme is more efficient, as we use binary addition which gives a padding system to RSA algorithm as a security analysis.

Keywords: Diffie Hellman key exchange, Discrete logarithm problem, Factoring assumption, Public key cryptosystem, RSA algorithm

A SIMILARITY SOLUTION FOR THE UNSTEADY COUPLED NAVIER STOKES-BRINKMAN EQUATIONS

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It is vitally important to have mathematical modeling for various industrial processes and natural phenomena such as gas diffusion, filtration, surface sublimation, and sedimentation. The design of the system is one of the most important steps during the modelling process that realistic models and appropriated boundary conditions. In this study, we present a self-similarity solution of the unsteady coupled Navier-Stokes-Brinkman equations for a laminar, incompressible flow that develops within a channel where the bottom of the channel is occupied by a porous media and the top of the channel is occupied by a free flow region with uniformly expanding walls. The governing equations of the motion are reduced to a fourth-order nonlinear differential equation and a Lie-group method is used to solve the nonlinear differential equation with the boundary conditions. The similarity solutions with respect to space and time are found. The effect of the Reynolds number (Re) and the dimensionless wall expansion ratio α have been studied analytically. Moreover, the velocity profile for different values of α is investigated. The stream-wise velocity profile of the fluid forces upstream in the head-end direction and later turns the head downstream towards the open end. The maximum normal velocity moves away from the wall when there is low injection into expanding channel.

Keywords: Lie-group method, Navier Stokes-Brinkman equation, Self-similarity solution

A NEW THIRD-ORDER APPROXIMATION FOR RIEMANN-LIOUVILLE FRACTIONAL DERIVATIVE AND APPLICATIONS

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Recently, Riemann-Liouville (R-L) fractional derivative of order α with $1 < \alpha < 2$ has been used to model anomalous diffusion in superdiffusive models in which the particles diffuse faster than in the regular diffusion, for example, superdiffusive continuous-time random walk models in the field of stochastic processes. Usually, the R-L derivative is approximated by the Grünwald approximation (GA). However, the GA has some limitations: it is of the first-order accuracy and displays unstable solutions for some numerical methods including implicit Euler (IE) and Crank-Nicolson(C-N) schemes. A shifted form of the GA is used to restore the stability for the IE and C-N schemes, whereas it is also of the first-order accuracy. Hence, it is of great importance to get high-order accurate numerical approximations for the R-L fractional derivative. Recently, a second-order accurate Grünwald type approximation with arbitrary shift was constructed for the R-L derivative and used to solve the space fractional diffusion equation with unconditional stability for C-N scheme. In this study, using the second-order approximation with non-integer shift, $r = (1 - 1/\sqrt{3})\alpha$, we establish a new third-order approximation for the R-L derivative and apply it for the fractional boundary value problem. This non-integer shift creates a misalignment of the grid points to intermediate points between the regular grid points. The misalignment is removed by shifting the point of the derivative to an intermediate point. Numerical results are presented to demonstrate the effectiveness of the approximation. The future aim of this work is to employ the new third-order approximation to seek numerical solutions of space fractional diffusion equations.

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Keywords: Generating functions, Riemann-Liouville fractional derivative, Second-order approximation, Shifted Grünwald approximation

A STUDY OF DENGUE CASES AND CORRELATION WITH CLIMATE FACTORS IN AKURANA, SRI LANKA

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Dengue is a major public health problem and a dominant vector-borne disease in Sri Lanka. Since Dengue is a local and focal disease, studies on the distribution of dengue cases with relevant climatic factors are essential for decision making on early warning of dengue outbreaks. The selected area for the study is Akurana, Kandy, which is in the Central Province of Sri Lanka. Dengue data was collected from the Ministry of Health (MOH) office, Akurana. Climatic data, which contains minimum temperature, maximum temperature, and precipitation, were collected from the weather station at Katugastota, the nearest to Akurana. The data set consists of weekly observations from 2014 to 2018. This study analyzed the intensive level of dengue cases over time using k -means. Cross-correlation is applied to identify the relationship between the number of dengue cases with stated climatic variables in eight-week lags. A Dynamic Linear Regression (DLR) Model was fitted to determine the association between climate factors and dengue cases with two-week lags. Three clusters were identified with the mean number of cases 0.31, 3.66 and 17.64, with each cluster containing 165, 83 and 14 weeks, respectively. Moreover, a positive correlation between dengue cases and the minimum temperature and a negative correlation between dengue cases and the maximum temperature were found. The precipitation was discarded in the use of the DLR model because they did not present a good correlation with the dengue cases within the time lags considered. The most critical climate variable which influences Dengue is the minimum temperature. Autoregressive Moving Average (ARIMA) and DLR models were compared, and the DLR has the least residual sum of squares (2611.9) compared to ARIMA (2736.6). Thus, DLR is useful when external information can be used in time series modelling. The study would be helpful to the stakeholders who have better insight into controlling the dengue epidemic in the area.

Keywords: Climate, Cluster, Dengue, Epidemic, Regression

NUMERICAL PREDICTION OF UNSTABLE LOCATIONS IN BRIDGE STRUCTURES

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Damage detection of bridge structures is important for bridge maintenance, which typically requires a huge amount of manual effort. In this work, an approach based on Proper Orthogonal Decomposition (POD) is introduced for predicting the unstable locations of bridge structures. POD is a dimension reduction method that can obtain a compact representation of high-dimensional data. Measured raw data can be reconstructed accurately using the dominant POD and therefore, in practice, removing low-level modes from a data set can eliminate inconsistent noise. In this study, the wireless sensor data is used as the training data for the approach. This study is conducted based on a bridge in Waddington, NY, USA. The bridge was instrumented in a rectangular grid at 30 locations with dual axial accelerometers. There are several diaphragms of the bridge and each diaphragm of the bridge is connected to the longitudinal girders through a series of six bolts. Damage tests are conducted by removing the 6 bolts in the 1st diaphragm connection. Three situations of the bridge are introduced for the study: baseline (healthy structure), damage 1 (removal of four out of six bolts), and damage 2 (removal of six out of six bolts). Each test was performed for ~90s by passing a truck three times in both directions of the bridge. Lateral and Vertical accelerations at sensors are measured and recorded at 128Hz after each pass of the truck. The accelerations of the damage 2 situation after the third pass of the truck are predicted using a finite-dimensional representation for the wireless sensor data. The most dynamic basis functions for the finite-dimensional approximation are selected using POD. The training data for the POD are the acceleration data of the baseline and damage 1 situation of the bridge after the third pass of the bridge. The prediction of the damage 2 situation after the third pass of the truck is obtained by projecting the acceleration data at the first and second pass of the truck onto the most dynamic POD modes. The predictions are in good agreement with the measured data and allow us to detect the unstable locations of the bridge. The results are compared using a statistical method based on kurtosis. Thus, this method is a promising non-invasive method for damage detection of Engineering structures such as bridges, beams, and buildings, etc.

Keywords: Acceleration, Bridge structures, Damage detection, Noninvasive method, Proper orthogonal decomposition

METHOD OF DIRECTLY DEFINING INVERSE MAPPING FOR BOUNDARY LAYER FLOW OF NANOFLUIDS

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Since late 20th century, homotopy perturbation and homotopy analysis methods are widely used to obtain analytical approximations for non-linear differential equations. If the operator of the non-linear differential equation contains a linear part and a non-linear part with a small parameter, then the original non-linear problem can be transformed into an infinite number of linear differential equations. The strategy of gain solutions to such a series of differential equations needs to find an inverse linear operator and, it is time-consuming. To overcome this obstacle, in 2016, Liao introduced the Method of Directly Defining inverse Mapping (MDDiM) to solve a non-linear ordinary differential equation with the freedom of choosing the inverse linear mapping directly, instead of calculating the inverse of an operator. Vajravelu and his research group recently extended the Method of Directly Defining the inverse Mapping (MDDiM) to systems of non-linear ordinary differential equations up to three-equation systems. In this study, we applied this novel method to solve coupled non-linear four-equation systems that arise in boundary layer flow of nanofluid with solutal concentration parameter. In fluid mechanics, a boundary layer is a layer of fluid near a bounding surface where the effect of viscosity is significant. The obtained results are compared with the numerical results and found to be a good agreement. Further errors of approximate series solutions are recorded and it is found to be very low. The equations discussed here deal with the triple diffusive boundary layer flow of nanofluid over a non-linearly stretching sheet. In this model, binary nanofluid is used, the Brownian motion, thermophoresis, and cross-diffusion are classified as main mechanisms and MDDiM leads the fastest convergence to the solution.

Keywords: Brownian motion, Cross diffusion, Nanofluid, Stretching sheet, Thermophoresis effects

AN OPTIMIZATION TECHNIQUE TO SCHEDULE THE LOCOMOTIVE DRIVERS IN A RAILWAY SUB-DIVISION IN SRI LANKA

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The service of a locomotive driver is of utmost importance to maintain the quality of service of the national transportation system. Scheduling turns of locomotive drivers (LDs) and assigning LDs according to the schedules are significant to have a smooth functioning in the transportation system. In general, the locomotive driver-turn assigning problem is fairly difficult to solve because of the irregular working shifts and the dearth of LDs. Also, there are many conflicts between drivers and the management who is responsible in assigning LDs to turns. This research work presents a scientific approach to address these issues by formulating a mathematical model so that the LDs will be assigned to turns while satisfying the requirements of the Kandy railway sub-division. The present driver-turn schedule in the Kandy sub-division lasts for a week and it is being repeated weekly. The main disadvantage of this scheduling is failing to assign uniform working hours among LDs. The procedure described here is an assignment model that finds a basic feasible solution to the zero-one integer linear programming problem that assigns LDs to a set of selected trains for a day with uniform overtime hours for each driver. To determine the optimal assignment, an objective function is formulated using a weighted method to minimize the overtime hours for each LD during the respective turn, where weights are assigned according to the overtime hours. It is assumed that the required LDs are available according to the schedule. The formulated model is solved using the Branch-and-Bound algorithm and the solution produces an optimum assignment, where the overtime is distributed uniformly with 6.35 hours for each LD during the scheduling time period.

Keywords: Branch-and-Bound algorithm, Scheduling problem, Zero-one integer linear programming

**GENETIC ALGORITHM FOR THREE BUYERS AND TWO SUPPLIERS
TRANSPORTATION PROBLEM WITH VARYING DEMAND AND SUPPLY**

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Transportation problem plays an important role in the logistics and supply-chain management for improving services by reducing cost and optimizing the use of resources. It is one of the most important and well-known problems which can be formulated as a linear programming problem. Researchers have given considerable attention to the transportation problem with fixed demand and supply. In literature, there are many algorithms to solve such problems. But, in real-world applications, demand and supply quantities may vary within a specific range due to variations in the global economy. Finding an exact upper minimal total cost of a transportation problem with varying demand and supply (TPVDS) is an NP-hard problem. Thus, less attention has been paid to this type of transportation problems. It can be observed that heuristic approaches have a higher potential in reaching a near optimal solution for transportation problems with varying demand and supply. Hence, in this study, a solution procedure based on the concept of genetic algorithm is proposed to solve three-buyer and two-supplier TPVDS. Our proposed algorithm provides a better near optimal solution than the two existing methods; Liu (2003) and Juman and Hoque (2014).

Keywords: Genetic algorithm, Linear programming, NP-hard problem, Transportation problem

ISOMORPHISM OF GRAPHS AND CLUSTERING BASED ALGORITHM FOR SIGNATURE VERIFICATION

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Handwritten signatures are being used in a wide variety of security applications. It is particularly important mainly as it is used in its entirety to verify an individual's identity in the legal, financial, and administrative areas. Therefore, there should be an adequate verification system of signatures to identify who is the signatory. In this study, an offline signature verification algorithm, which uses geometric features, is used to verify a person's signature. The algorithm consists of two steps, which use the mathematical concepts of isomorphism of graphs and clustering of the vertices of the signature. In the first step, the relative percentage of vertices of forgeries is calculated and forgeries are identified using a threshold number, which is predetermined based on genuine signatures. In the second step, the forgeries that could not be identified in the first step are identified using the optimum number of clusters of vertices and the centroids of each cluster of genuine and forged signatures. The clusters of the vertices are generated using the k -mean clustering algorithm. The results show that the proposed algorithm verifies forgeries with a success rate of 68%. Thus, the proposed identification method is a very promising approach in the field of handwritten signature verification.

Keywords: Forgeries, Genuine signature, Isomorphism of graphs, k -mean clustering, Off-line signature verification

A MODIFIED ELGAMAL CRYPTOSYSTEM USING THE TRAPDOOR KNAPSACK ALGORITHM

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The ElGamal cryptosystem was introduced by Taher ElGamal in 1985. It is one of the most widely used public key cryptosystems and a probabilistic algorithm that was developed based on the Diffie-Hellman key exchange protocol. Unlike the Diffie-Hellman algorithm, this is a complete encryption-decryption system that depends on the discrete logarithm problem. Its security is based on the difficulty of finding the discrete logarithm modulo a large prime number. The new trapdoor-knapsack public key cryptosystem's encryption equation is based on the general modular knapsack equation, but unlike the Merkle-Hellman scheme, the knapsack components do not have to have a super increasing structure. The knapsack nature of the system ensures that the efficiency of encryption and decryption is higher when compared with RSA-cryptosystem. Incorporating these hypotheses, we have developed a modified ElGamal algorithm using trapdoor knapsack algorithm as a combination of the standard ElGamal algorithm and the new trapdoor knapsack algorithm. The key generation and the decryption process of the new scheme is different from the standard ElGamal algorithm. In this new scheme, the plaintext is taken into its binary form and also, we considered two prime numbers. The decryption process carried out in two steps; first, the general modular knapsack equation is recovered and next the binary form of the plaintext is obtained using the private knapsack vector and the additional prime number. The security of the new system depends on the discrete logarithm problem which is known to be computationally hard and also the system preserves security against the chosen plaintext attack.

Keywords: Chosen plaintext attack, Diffie-Hellman key exchange, Discrete logarithm problem, ElGamal cryptosystem, Trapdoor knapsack algorithm

A STUDY ON THE GOLDEN RATIO AND THE GENERALIZED FIBONACCI AND LUCAS NUMBERS

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Conferred to the interesting relations and applications not only in mathematics, but also in nature, art and architecture, the Golden ratio, φ has intrigued mathematicians throughout the history of mankind. It is closely related to two famous additive sequences given by the recursive relation $A_n = A_{n-1} + A_{n-2}$ which requires two seed values; if the initial values A_0 and A_1 are 0 and 1 respectively, we call it the Fibonacci sequence $\{F_n\}$, and if the values are 2 and 1 respectively, it is the Lucas sequence $\{L_n\}$. It has been proven that the ratios of two consecutive terms of these sequences will converge to φ as n increases. Robert Schneider, in his expository paper titled “Fibonacci Numbers and the Golden ratio”, has generalized the quadratic polynomial for the Golden ratio and has modified the Fibonacci sequence accordingly. The Golden ratio in the form $\varphi(a, b)$ is defined as the largest root of the quadratic equation $x^2 - ax - b = 0$, and the corresponding Fibonacci sequence has been obtained as $F_n(a, b) = aF_{n-1}(a, b) + bF_{n-2}(a, b)$ with initial values $F_0(a, b) = 1$ and $F_1(a, b) = a$. We have studied some unexplored areas in literature through the suggestions made by Schneider and obtained similar results to generalize the Lucas sequence as $L_n(a, b) = aL_{n-1}(a, b) + bL_{n-2}(a, b)$ with $L_0(a, b) = 2$ and $L_1(a, b) = a$. Continued fractions were used to define the ratios $\frac{F_n(a,b)}{F_{n-1}(a,b)}$ and $\frac{L_n(a,b)}{L_{n-1}(a,b)}$ and, it was observed that these ratios converge to $\varphi(a, b)$ regardless of the initial values as n increases. However, numerical computations revealed that the rate of convergence differ for different initial values with the slowest rate given to $F_0(a, b) = 0$ and $L_0(a, b) = 0$. The ratios are undefined when $F_1(a, b) = 0$ and $L_1(a, b) = 0$. Finding similar patterns associated with the Silver ratio and Pell Numbers is our immediate next task. Also, as future directions, one can work with higher degree polynomials (i.e., degree k polynomials for $k > 2$) and explore more on the properties of associated $\varphi(a_1, a_2, \dots, a_k)$ and $F_n(a_1, a_2, \dots, a_k)$.

Keywords: Continued fractions, Fibonacci sequence, Golden ratio, Lucas sequence

ON THE RADIUS OF CONTRACTION FOR THE NEWTON'S MAP

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Newton's method or Newton-Raphson method, which was named after Isaac Newton and Joseph Raphson is a technique of finding roots of a polynomial which produces successive better approximations by applying the concept of iterative functions. Specifically, it involves choosing an initial guess x_0 , and then through the iterative process, finding a sequence of values x_0, x_1, x_2, \dots that converges quadratically to an exact solution. The set of points which converge to a particular root is called a *basin of attraction*. Not only for real-valued functions, but also this method generalizes to complex-valued functions as well. If we wish to find the value x which satisfies the equation $f(x) = 0$, in this method we guess some initial value x_0 close to the desired solution and then get a better approximation: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$. The existing result for the *radius of contraction* of Newton's method is $\Delta(\alpha, r) \subseteq A_f(\alpha)$. Here $\Delta(\alpha, r) = \{z \in \mathbb{C} : |z - \alpha| < r\}$ is the open disk with centre α and radius r and $A_f(\alpha) = \{z \in \text{domain}(f) : f^n(z) \rightarrow \alpha\}$ is the basin of attraction of α for a complex-valued function f mapping its domain set into itself. In our work, we found a stronger statement for radius of contraction, r than obtained by $\Delta(\alpha, r) \subseteq A_f(\alpha)$. The value $r = d \frac{(2k-1)}{(2n-1)}$ found in this research is for the Newton's method at an iterative root α of the polynomial $p(z) = (z - \alpha)^k(z - \alpha_1) \dots (z - \alpha_s)$ of degree $n = k + s$, where α_j need not to be distinct and $d = \min_{j=1,2,\dots,s} |\alpha - \alpha_j|$.

Keywords: Basin of attraction, Iterative root, Newton's method, Radius of convergence

RIGHT CENSORING DATA WITH PARAMETRIC AND NON-PARAMETRIC ESTIMATIONS

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Right censoring arises naturally in lifetime data. It happens when a subject withdraws from the study before an event happens, or when the study ends before the event happened. All units under study will not be followed until they fail and the examiner may have to stop at a certain time while some units are still in operation. This introduces right censoring into the data. Some commonly used lifetime distributions are Weibull, Exponential, Lognormal and Gamma. From the present study we wanted to identify from which distribution our application with right censoring data comes from. Because of that our study deals with the parameter and non-parameter estimation. First, we consider the maximum likelihood estimations (MLE) of Exponential and Weibull distributions. Exponential distribution has one parameter. Therefore, we obtained one maximum likelihood estimate. However, Weibull distribution have two parameters. Thus, we computed two estimates. Afterwards, we computed the confidence limit for those parameters. Then we plot survival curves according to the parametric estimation. Subsequently, we obtained two curves corresponding to the Exponential and Weibull distributions. We used the Kaplan-Meier (KM) estimator as a non-parametric estimation for survival analysis. By using RTM mathematical software, we calculated this KM estimate. Finally, we plotted the Kaplan-Meier survival curve with the confidence bounds and compared the curves. We took non-parametric estimation method to confirm our parametric estimation. According to the curves, Weibull and Exponential distributed survival curves overlapped with the KM estimate curve. Therefore, we can conclude that data is coming from the Weibull distribution or otherwise from the Exponential distribution.

Keywords: Censored data, Kaplan-Meier, Maximum likelihood

**SOLVING TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS
USING ZERO - POINT METHOD COMBINED WITH MORE – FOR - LESS
PROCEDURE**

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The Transportation Problem (TP) is a special class of linear programming problem that deals with transporting a homogeneous commodity from sources to destinations. The main objective of the TP is to find the best possible way to ship commodities with minimal cost of transportation. In real world, most of the TPs consist of mixed constraints, where a specially designed method is needed to solve them. In the literature, it has been proven that the Zero-Point method (ZPM) is capable of solving TPs with mixed constraints and it gives either a better initial basic feasible solution or the optimal solution in many instances. Thus we can conveniently apply this method to real world TPs. Moreover, there is a paradox called More-For-Less procedure, more products for less cost, which can be used to obtain a better near optimal solution or global optimal solution if one exists. Basically, this procedure is based on the shadow prices in the cost matrix. So practically this method is useful to a decision maker in determining which plant capacities are to be increased and which markets should be sought. In our study, the proposed method gives an optimal transportation schedule for a specified period of time. This process can be used in a distribution sector which has a scheduled distribution pattern in the markets. The proposed method is less complicated and less time consuming in reaching a near optimal or the optimal solution. However, in certain cases the ZPM tends to produce degenerate solutions. The proposed method is illustrated using a randomly generated TP with the help of Monte Carlo simulation. It can be concluded that the proposed method is competitively effective in preparing optimal transportation schedules satisfying the requirements of supply and demand in the best possible manner.

Keywords: Mixed constraints, More-for-less procedure, Optimal solution, Transportation problem, Zero-point method

MARIGOLD GRAPH AND ITS NEW LABELLING METHOD SIMILAR TO THE ANTIMAGIC LABELLING

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The initial idea of antimagic labelling was introduced in 1989 by Hartsfield and Ringel. It is defined as follows: consider a graph G with an m number of edges and n number of vertices. The anti-magic labelling of G is a bijection from the set of edges to the integers $\{1, \dots, m\}$ such that all n vertex sums are pairwise distinct. In this study, we introduce a new graph called the Marigold graph and a new labelling method similar to the anti-magic labelling. The Marigold graph is generated from any number of copies of fully binary trees which are going through concentric circles. All copies of trees are connected to a middle vertex and the height of the Marigold graph is increasing with concentric circles. One copy is considered as one petal in the marigold graph. A Marigold graph with n copies (petals) and height (number of concentric circles) k is denoted by M_k^n . The new labelling method which is similar to the antimagic labelling is defined as follows: a graph with m edges and n vertices is labelled as an injection from the set of edges to the integers $\{1, \dots, x\}$ (where x is any integer) such that all n vertex sums are pairwise distinct, where the vertex sum is the sum of labels of all edges incident with that vertex. In this study, for edge labelling we consider the petals one by one and denote the r^{th} edge at k^{th} level as e_r^k , and define a function to label edges of the first petal. Then define the new labelling method for other petals, such that for the n^{th} petal, edge labelling is starting with $J_{n-1} + 1$ (where J_{n-1} is the summation of all edge values in the $(n - 1)^{\text{th}}$ petal, $\sum_{i=1}^m e(n - 1, i) = J_{n-1}$) and continue the labelling as a monotonically increasing sequence. Finally, it can be concluded that we can label the Marigold graph according to the new labelling method similar to the anti-magic labelling.

Keywords: Anti-magic labelling, Complete binary trees, Concentric circles, Marigold graph

ANALYSIS OF AIRBORNE MICROBES FROM SEQUENCING DATA

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The interest in the role of airborne microbiotas in built environments with regards to health and disease has been rapidly growing, especially since the emergence of the on-going coronavirus pandemic. A deeper understanding of the dispersal and transmission of airborne pathogens can be gained through monitoring bioaerosol at high spatiotemporal resolution using a portable, field-deployable bioaerosol sampler. The device is designed for an efficient collection of particles in the size range of 0.01 – 10 μm , thus making it suitable to capture diverse microbial populations including bacteria, fungi, and viruses. We deployed these devices in a hospital at key indoor locations and samples were collected periodically to conduct next-generation sequencing (16S rRNA and ITS) from the extracted DNA. Downstream analysis of sequencing data aimed to capture the trends in bacterial and fungal diversity across different seasons and locations within the hospital. Visualization via t-Distributed Stochastic Neighbor Embedding revealed that microbial compositions were relatively homogenous between locations but were heterogenous over time. In addition, clustering of differentially abundant pathogens identified associations between certain nonpathogenic bacterial genera with pathogens known to cause healthcare-associated infections such as *escherichia*, *shigella*, and *ralstonia*. Bioaerosol sampling in conjunction with these analytical techniques could potentially be applied for both routine monitoring of the general built environment and detection of specific threat due to airborne pathogen.

Financial assistance from the Lawrence '57 and Antoinette Delaney Ignite Research Fellowship is acknowledged.

Keywords: ANCOM, Bioaerosol, Microbiome, Nosocomial infection, tSNE

AN INVESTIGATION OF SPACES WITH ASYMPTOTIC DIMENSION

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The topological dimension is a fundamental concept in topology. It was a concept that was under discussion in the early 20th century and continued to enrich the modern field of dimension theory. The covering dimension is one of the many different topological definitions of dimension and is arguably the most used definition. The covering dimension is defined as the least integer n , for which any open cover of a topological space X has a refinement of order n , and we say the space is infinite-dimensional if no such n exists. Large scale geometry borrows ideas from topology and coarsens them to define invariants such as the asymptotic dimension. The asymptotic dimension, $\text{Asdim}(X)$ is the large scale version of the topological dimension. The definition is just a coarsening of Lebesgue's covering dimension. In this study, we look into a definition of topological dimension as a result of the theorem of partitions by Eilenberg and Otto. We coarsen this definition to define a version of asymptotic dimension that we call separator-based asymptotic dimension, $\text{Asdim}_s(X)$. We show that for discrete groups \mathbb{Z}^n , the two definitions agree. The upshot of the separator-based asymptotic dimension is that it allows us to define different classes of infinite-dimensional spaces analogous to strongly and weakly infinite-dimensional spaces in classical dimension theory.

Keywords: Asymptotic dimension, Covering dimension, Large scale geometry

REDUCED ORDER MODELING FOR WEATHER FORECASTING

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Weather forecasting means determining the state of the atmosphere for a particular region using science and technology approaches. It is one of the most challenging problems in the world. Historical weather data, which can be used for weather prediction, is rich in patterns with helpful information. In this work, we develop a reduced order model (ROM), which is usually thought of as computationally inexpensive mathematical representations that offer the potential for near real time analysis, for weather prediction in Sri Lanka. For this work, we consider five most affected environmental variables for weather prediction in Sri Lanka: atmospheric pressure, relative humidity, average wind speed, average temperature, and precipitation. Twenty-four Sri Lankan sensor stations are used for the analysis. Proper Orthogonal Decomposition (POD), which is a dimension reduction technique, is used to transform eigenvectors of the data covariance matrix of historical weather data to a new coordinate space defined by the basis that have a minimum variation of the weather data. The most dynamics basis functions, which are generated using POD from historical weather data, are used to create a ROM for the original problem. The ROM is capable of reconstructing the weather factors successfully at a given sensor, which did not use for the basis function generation. The accuracy of the results is measured using the root mean square error. We showed that the error of the predicted precipitation for 100 consecutive days at the Galle sensor station is $1.47 \times 10^{-12}\%$. This approach is an excellent candidate for weather forecasting and early detection of natural disasters. The future studies focus on comparing the POD approach with existing weather forecasting methods.

Keywords: Proper Orthogonal Decomposition, Reduced order model, Weather forecasting

CONNECTIVITY OF GRAPHS CONSTRUCTED BY GRAPH OPERATIONS ON $K_{m,m}$

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During the last decade, graph theory has attracted many researchers. In Mathematics, the meaning of connectivity is one of the fundamental concepts in graph theory. It has prominent applications such as finding network tolerance of a graph, brain connectivity, network tolerance of routing transportation networks. It demands a minimum number of elements (nodes or edges) that require to be removed to isolate the remaining nodes into separated subgraphs. In graph theory, a connected graph G is said to be k -vertex-connected (or k -connected) if it has more than k vertices and remains connected whenever fewer than k vertices are removed. A complete bipartite graph is a graph whose vertices can be partitioned into two disjoint subsets in such a way that no edge joins two vertices in the same set. Our work is mainly focused on developing theoretical concepts such as finding general formulas for connectivity of resulting graphs obtained by using mathematical operations of summation and multiplication. Under that, we looked for various connectivity properties of complete bipartite graphs. The multiplication operation over a complete bipartite graph is denoted by $*G$ and is defined as $K_{m,m} * G = K_{l,l}$. The product of n copies of $K_{m,m}$ is denoted by $(K_{m,m})^n$. Then the edge connectivity $\lambda(G)$ of $(K_{m,m})^n$ is $\frac{(2m)^n}{2}$ and it is proved by using a combinatorial proof. Also, by using the principle of mathematical induction, it is proved that the edge connectivity of finite summation of n copies of $K_{m,m}$ is $m(2n - 1)$, where m is the number of vertices in one partite set or degree of one vertex and n represents the number of copies of $K_{m,m}$. The study can be further extended by finding general connectivity formulas for other standard graphs and applying these connectivity concepts for real-world applications such as finding network tolerance, brain connectivity, routing planning networks, etc.

Keywords: Complete bipartite graph, Edge connectivity, Graph operations

MATHEMATICAL MODELING OF EVOLUTION OF COVID-19 DISEASE

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Mathematical modeling can be used to understand the dynamics of transmission of infectious diseases like COVID-19, SARS, Ebola, and Dengue among populations. In this work, a one prey-two predator model has been developed to understand the underlying dynamics of COVID-19 disease transmission. In the one prey-two predator model, both predators predate the prey but no predation process takes place between the two predator populations. We identified three populations; infected, recovered, and dead, which can be used to understand the transmission of COVID-19. Since the infected population gets transmitted to both recovered and dead populations, the two prey-one predator model was used to build the model related to COVID-19. The existence and uniqueness of the solutions for the model were proved and the conditions for the local and global asymptotic stability of the equilibrium points were established. Local asymptotic stability of the vanishing equilibrium point, recovered and death-free equilibrium point, recovered population-free equilibrium point, and the death-free equilibrium point are proved under some conditions. Global asymptotic stability of the recovered population-free equilibrium point and death-free equilibrium point are also analyzed. Moreover, the model parameters for the model were estimated using the Nelder-Mead optimization method for a data set that consist of the total number of infected, recovered, and dead population in 2020. With the continuation of time, the infected population converges to a constant value, the recovered population declines and reaches zero, and the death population attains a constant value. However, some modifications to the system are needed. In future work, measures such as health precautions, vaccinations are needed to be considered for the formulation of the model.

Keywords: COVID-19, Equilibrium, Mathematical modeling, Parameter estimation, Stability

MATHEMATICAL MODELING OF BLOOD FLOW THROUGH DISEASED ARTERIES**P.R. Sandadini^{*} and U.G.I.G.K. Udagedara**

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Blood clots, plaques, and swell are diseases that occur in the arterials. The *Atherosclerotic plaque* builds up due to the overtime fat and the calcium deposition may increase and build up inside the lining of the artery wall. This can lead to narrowing of the artery wall and blood flow through it will become restricted. Then, the volumetric flow rate will reduce and lead to blockage of the artery. When atherosclerosis affects the arteries, which supply blood to the heart, restriction of the blood supply to the heart muscle can occur and lead to a heart attack. This study mainly focuses on the modeling of blood flow through diseased arteries. To build a model for this kind of arterial, the velocity distribution through a free flow channel covered by porous media is considered. The motion of red blood cells through the free flow region covered by the porous media is modeled by considering blood flow as a free flow and any kind of disease, mainly the artery's swell as porous media. Here, a very thin cylindrical annulus is considered as an artery. The inner cylinder is filled with the red blood flow and the outer cylinder is filled with plaque. The quantitative analysis is restricted to Newtonian fluid flow with a diameter of the $2L$ inner cylinder. The thickness of the upper layer and the bottom of the layer to the cross-section of the artery are taken as H_1 and H_2 , respectively. Due to the swell of the artery or any kind of disease, the thickness of the outer cylinder is taken to be different. Porosity ε and permeability K can also be different in the two layers of the outer cylinder. The equations governing the motion of the regions are coupled with the Navier-Stokes Equation and Brinkmann Equation. The velocity distribution of the diseased arteries is found by solving the governing equations for the motion. The results show that the maximum velocity occurs in the free-flow region. Parabolic velocity profile, which can be seen in the free-flow region, no longer exists in the porous media due to the effect of the porosity.

Keywords: Blood flow, Brinkmann equation, Navier-Stokes equation, Newtonian fluid, Porous material

**CORRECTION MODELS FOR ACCURATE PM_{2.5} ESTIMATION FROM
LOW-COST AIR QUALITY SENSOR DATA**

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Environmental epidemiology requires accurate estimates of pollutant levels in ambient air to evaluate the health effects from long-term exposure. Among these pollutants, particulate matter (PM) with a diameter of 2.5 microns or less (PM_{2.5}) has been shown to associate with various acute and chronic health problems. In United States, Environmental Protection Agency (EPA) provides reliable measurements of PM_{2.5} but the impact is limited by a sparse distribution of monitoring sites. Air quality measurements using low-cost sensors such as Purple Air (PA) are currently considered to overcome this limitation. However, measurements from these sensors could be noisy, and an appropriate correction model will be required to get an accurate estimate of PM_{2.5}. In this work, we aim to improve upon the correction model for PA sensor-derived PM_{2.5} data using EPA data as a gold standard with identifying the effect from distances between EPA and PA for the models. In total we have collected hourly PM_{2.5} measurements from 14 PA sensors located at Cook County in the Chicago area from August 2019 to July 2020 and developed a modeling approach that uses multiple regression techniques together with distance analysis. According to our results, the temperature was more influential ($R^2 = 0.60 \sim 0.80$) factor for correction models compared to the relative humidity ($R^2 = 0.50 \sim 0.60$). In addition, the distance between the PA sensors and the EPA sites did not improve the correction models, however the correction model with multiple PA sensors yielded better model performance than the model with individual PA sensor. These correction models along with data pre-processing techniques implemented in this work could be promising in successful air quality administration.

Keywords: Environmental Protection Agency, Particulate matter, Purple Air sensors

ON REMOVING A WEAKLY INFINITE-DIMENSIONAL SUBSPACE FROM A HILBERT CUBE MANIFOLD

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Hilbert cube manifolds (or Q -manifolds) are separable metric spaces that admit open covers by sets homeomorphic to the Hilbert cube, $Q = [-1,1]^\infty$. Hilbert cube manifolds are ubiquitous in infinite-dimensional topology as they embody many characteristics of finite-dimensional manifolds. There are many different classes of infinite-dimensional spaces in dimension theory, such as strongly infinite-dimensional spaces, weakly infinite-dimensional spaces, and countable dimensional spaces. A topological space X is said to be weakly infinite-dimensional if for every family $\{(A_i, B_i): i \in \mathbb{N}\}$ of pairs of disjoint closed subsets of X , there exist separators D_i between A_i and B_i such that $\bigcap_{i=1}^\infty D_i = \emptyset$. Weakly infinite-dimensional spaces share some characteristics with finite-dimensional spaces, thus are of interest in dimension theory. In 2015 it was proved that the Hilbert cube could not be separated by removing a weakly infinite-dimensional subspace. This result was generalized in 2017 to show that the complement of the weakly infinite-dimensional closed subspace in Hilbert cube is acyclic. In this study, the acyclicity result was extended for Hilbert cube manifolds. We show that if X is a Q -manifold and $Y \subseteq X$ is a closed, weakly infinite-dimensional subspace, then $\tilde{H}_i(X \setminus Y) \cong \tilde{H}_i(X)$ for all i . That is, the reduced homology of a Hilbert-cube manifold will not change if a closed, weakly infinite-dimensional subspace is removed. This result can be mainly used in proving that the complement of a σ -compactum in the Hilbert Cube is Steenrod acyclic in dimension zero and might be instrumental in proving that the same result is valid in all the other dimensions.

Keywords: Hilbert cube manifold, Homology, Weakly infinite-dimensional spaces

MATHEMATICAL MODELLING OF ENVIRONMENTAL TRANSMISSION OF CORONAVIRUS

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Coronavirus disease 2019 (COVID-19) is caused by a virus which is called as Severe Acute Respiratory Syndrome Coronavirus 2 (*SARS-CoV-2*). It was identified in 2019 as the cause of an outbreak of COVID-19, first detected in Wuhan, China. It is mainly transmitted to humans from direct contact with respiratory droplets produced by a COVID-19 infected person. Also, the coronavirus can survive on outside surfaces for some time and hence, COVID-19 can be transmitted by touching objects and surfaces on which the droplets are landed, and then touching the eyes, nose or mouth. In our study, we developed two compartmental models to describe the dynamics of the COVID-19 when susceptible individuals stay in a smaller contaminated closed area for a shorter time period. Here, we study the dynamics of the COVID-19 prevalence by taking only the environmental-to-human and human-to-human transmissions into account with the threshold level of *SARS-CoV-2*, which is required to result a COVID-19 infected person. In this study, we proposed a function to represent this threshold level of the virus to result in an infection and we used this function to construct two mathematical models to find the spread of coronavirus via direct human-to-human contacts and indirect environment-to-human contacts in a small area.

The financial assistance from the PGIS Research Grant 2020 (PGIS/2020/04) is acknowledged.

Keywords: COVID-19, Environmental transmission, *SARS-CoV-2*, Threshold level

**USE OF ACTIVITY BASED APPROACH IN TEACHING GEOMETRY IN G.C.E. (O/L):
A CASE STUDY IN KALKUDAH EDUCATIONAL ZONE**

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Geometry is one of the key components in Mathematics curriculum in G.C.E. (O/L). According to the past G.C.E. (O/L) examination results, students' performance in Geometry has not reached the expected level. This study focused on the use of activity-based learning to instruct Geometry for G.C.E. (O/L) students and to evaluate its effect on students' performance. The teachers' and students' attitudes towards activity-based learning are also investigated. The mixed methods approach was used for the study. As the study sample, teachers were selected from schools irrespective of the type, but the students were selected purposively from 1AB and 1C schools in Kalkudah Educational Zone. Attitude tests for both teachers and students were carried out to identify their outlooks towards Geometry. Two groups of students were grouped into control and experimental groups based on their latest term test marks. Lesson plans with the activity-based approach were developed for the experimental group. The traditional teaching method was adopted for the control. At the end of the intervention, a test was conducted for both groups and performance was analyzed using the independent sample *t*-test. The results confirmed that the experimental group performed better than the control group (*p*-value was 0.23, which is greater than the alpha level of 0.05). Despite the fact that approximately 75% of teachers claimed that teaching Geometry is not a difficult task, students have performed poorly. Based on the results of the study, we strongly recommend to implement the activity-based learning method for teaching Geometry in G.C.E (O/L) classes.

Keywords: Activity-based learning, Geometry, Kalkuda zone, Mathematics

**FACTORS INFLUENCING STUDENT SELECTIONS IN PHYSICAL SCIENCE
STREAM AT G.C.E. (A/L): A CASE STUDY OF
KALKUDAH EDUCATIONAL ZONE**

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This study was designed to identify factors affecting students' selection of Physical Science stream at the G.C.E. (A/L). The study was conducted using seven selected schools in the Kalkudah Educational Zone. The sample consisted of Physical Science Stream students (58) and non-Mathematics Stream students (212), all G.C.E. (A/L) teachers (198), Zonal Education Officers (06), and Principals of 1A/B schools (05). A mixed methods approach was employed. The quantitative data were gathered through survey questionnaires and analyzed using SPSS and MS Excel. The qualitative data were obtained from structured interviews and analyzed using thematic analysis. The results indicated that students selected Physical Science Stream mainly out of their self-interest, their ability to learn Mathematics and wishful career opportunities. It was also revealed that families' economic status, poor results for Mathematics at the G.C.E. (O/L) Examination, lack of facilities at schools, perception of Mathematics as a difficult subject, lack of additional classes for Mathematics, influence of peers, misunderstanding of higher education and career opportunities are some of the reasons for not selecting Physical Science Stream. Grade five scholarship achievement has also been identified as one of the indirect factors for selecting Physical Science Stream at G.C.E. (A/L). Moreover, the gap between subject contents of G.C.E. (O/L) and G.C.E. (A/L) Mathematics demotivated students selecting Physical Science Stream. Some students with poor economic background have chosen Physical Science Stream out of their self-motivation and willingness of learning Mathematics. It is important to provide students some support through career guidance workshops to enhance their motivation and enthusiasm in selecting Physical Science Stream at G.C.E. (A/L).

Keywords: Affecting factors, Physical Science Stream, Self-Motivation, Students' perception

PERCEPTIONS OF UNIVERSITY ACADEMICS ON THE USE OF ADVANCED MATHEMATICAL KNOWLEDGE IN TEACHING ‘LIMITS’ IN COMBINED MATHEMATICS

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The present study reports on the required Advanced Mathematical Knowledge (AMK) in teaching the concept of ‘Limits’ in Combined Mathematics curriculum at G.C.E (A/L). Despite the popular agreement that it is beneficial for teachers to have some knowledge in advanced mathematics that they gain at the university, little progress is seen in connecting AMK explicitly in teaching practice in the Mathematics Education literature. This is part of an ongoing study that aims at exploring the use of AMK in teaching Combined Mathematics in Sri Lanka. In the first stage of the study, five senior university academics representing five different universities of Sri Lanka who have worked closely with teachers in G.C.E. (A/L) Combined Mathematics were given an open-ended questionnaire to find out their perceptions of AMK in teaching the concept ‘Limits’ at G.C.E. (A/L). A grounded theory was adopted for the analysis of data. The results showed both *what* kind of AMK is required and also *how* it can be used in teaching ‘Limits’ at G.C.E. (A/L). The following five categories were identified that show how AMK in relation to ‘Limits’ can be used in teaching: to provide correct intuition, as justification for results, for graphical explanations, to resolve misconceptions and to derive certain results. Even though AMK is often perceived by teachers to have no direct relevancy to classroom teaching, this study shows that AMK in fact could be transformed, while preserving the mathematical precision of the concepts, into a version that is useful to be directly used in teaching to give a precise and accurate understanding of difficult concepts such as ‘Limits’. This possible transformation of AMK into pedagogical content knowledge provides some insight to fill the gap in research on connecting AMK to pedagogy.

Keywords: Advanced Mathematical Knowledge (AMK), Calculus, Limit concept in Mathematics

**USE OF TECHNOLOGY BY PROSPECTIVE TEACHERS IN
PRIMARY SCHOOL MATHEMATICS**

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Mathematics helps individuals to perform well in today's society. It is essential to develop abstract concepts of Mathematics at an early age. Use of technology makes easy to develop mathematical concepts in the teaching-learning process. The aim of this study was to introduce the use of technology in teaching-learning process of Mathematics in the Primary Education to enhance the use of technology in students' meaningful learning. This is action research accomplished with the second-year prospective teachers of a National College of Education in Sri Lanka. The sample consisted of 36 prospective teachers who follow a primary education course and 72 Grade 1 students. Data were collected through questionnaires, observations, documents and interviews. Television, overhead projector, computer, smartphone and multimedia were introduced in the teaching-learning process as interventions. After three interventions, perceptions were gathered from prospective teachers and students. Quantitative data were analyzed using MS-Excel and qualitative data were subjected to thematic analysis. The findings suggest that the introduction of technology in the teaching-learning process of Mathematics among primary students, the teachers' enthusiasm enhanced, and motivated and engaged students in lessons. Thus, it is crucial to encourage prospective teachers in using technology in teaching Mathematics in order to enhance the satisfaction of teaching-learning process among primary level students.

Keywords: National College of Education, Primary mathematics, Prospective teachers, Technology

**DEVELOPMENT OF TRIGONOMETRIC FUNCTIONS WITH EDEXCEL
G.C.E. (A/L) STUDENTS: A CASE STUDY IN KANDY EDUCATIONAL ZONE**

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Trigonometry is an important concept in Mathematics. However, when students begin to study Advanced Level (A/L) Mathematics, they find it difficult to apprehend basic trigonometric identities. It is more critical with students who possess average mathematical skills. As such, this study focused on improving (A/L) Mathematics students' performance in trigonometric relations. The sample consisted of 52 students who are following A/Ls in Edexcel curriculum. The qualitative and quantitative data were collected. Quantitative data were analysed using MS-Excel and the qualitative data using thematic analysis. The first intervention was to find out the importance of Trigonometry for day to day life. The second intervention was based on observations in the first phase of the intervention. The final intervention was a panel discussion on the importance of Trigonometry in real-world applications. The implementation of all three interventions have resulted in developing students' motivation, enthusiasm, and soft skills. The findings indicate that after the interventions, the students find it more comfortable in answering the questions on Trigonometry. Moreover, students have developed confidence in initiating novel projects based on Trigonometry. Thus, it is suggested to re-evaluate classroom practices in teaching mathematics to enhance students' meaningful learning as 'preparing today can save us from repairing tomorrow'.

Keywords: GCE Advanced Level, Panel discussion, Trigonometry identities

STUDENT ERRORS AND MISCONCEPTIONS ASSOCIATED WITH FRACTIONS: POSSIBLE CAUSES AND PROBLEMATIC CONCEPTIONS IN ADDITION AND SUBTRACTION

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The creation and comprehension of the concept of fractions is time consuming. Therefore, fractions are one of the areas in primary mathematics, which both teachers and students find difficult. Studies show that students have shown difficulties in learning the concept of fraction at all Grades. The objectives of this study were to identify the errors in performing addition and subtraction of fractions, investigate the causes that led to these errors, and finding possible ways that can relieve students' misconceptions and errors. The study was conducted for Grade 9 students of four secondary schools in Nuwara-Eliya district. The sample consisted of 120 students selected randomly from Grade 9 based on the marks obtained at the term test. The sample also included all six teachers who teach mathematics for Grade 9. The students were assigned class work, homework, and an in-class test, consisting of questions designed to address different concepts and misunderstandings on addition and subtraction of fractions. At the end, randomly selected students from the group were also interviewed. Prior to interviewing the students, the six teachers completed a questionnaire, which contained questions to supplement the interviews. The answer scripts were examined in three categories as correct, incorrect and blank and solutions belong to incorrect category were examined in detail and mistakes were coded and the reason of these mistakes were dwelled on. Based on the survey, many students made conceptual, procedural and careless errors and remedial measures were suggested to overcome them. The Chi-squared test was performed to determine whether there is an association between these three types of tasks. Based on the results, there is an association between the errors in the examination task and the class work task, but no association with the homework task. Thus, current practices in teaching and learning process have to be reevaluated for meaningful learning in Mathematics.

Keywords: Concepts, Errors, Fractions, Misconceptions

**WINNING A GOLD MEDAL IN INTERNATIONAL MATHEMATICAL
OLYMPIAD: A COMPARATIVE STUDY**

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International Mathematical Olympiad (IMO), a competitive gathering of individuals with prolific mathematical intelligence, has garnered much interest in analyzing inter- and cross-country differentiation of Mathematics Education. This study attempts to quantify the relationship of mathematical knowledge and population indices of a country, based on the performance at IMO and to recognize the most contributing indices that promote Mathematics Education. In this study, 2214 observations were used along with 202 indices such as educational, welfare, health, population etc. collected from 107 countries over the period 1984-2017. The analysis is conducted with the application of Penalized Logistic Regression methods, Logistic Regression, Support Vector Machines (SVM) and Random Forest (RF) methods. Stratified hold-out cross schema was used for the validation process and the performance of fitted models was evaluated using accuracy, specificity, sensitivity, and F1-score metrics. Contributing factors that provide or hinder Mathematics Education, which highly affect the performance of the model, were gathered based on the area ROC statistic as a result. The RF model with engineered features illustrated significant performance metrics with 91.86% accuracy and 0.9188 F1-score and was prioritized in identifying highly contributing indices. A significant number of indices were affiliated with education (Secondary and Primary education). Expenditure on tertiary education and school enrolment (primary, secondary, tertiary) contributed positively to the model and health-related indices such as number of Physicians, sanitation services, measles vaccinations and hospital utilities contributed moderately yet positively towards model predictions. Furthermore, the results of this study showed that the aforementioned indices have a high influence on the fitted model which concludes that by advocating and improving those indices that promote or hinder Mathematics Education respectively, a nation can stand to benefit from consequential scientific growth.

Keywords: Classification, Elastic Net, International Mathematics Olympiad, Mathematics Education, Random Forest

**INVESTIGATION OF WAYS OF TEACHING COORDINATE GEOMETRY: A
CASE STUDY IN COLOMBO AND GAMPAHA DISTRICTS**

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Geometry plays a significant role in Mathematics and improves the logical thinking power of the human brain. However, Students' performance in the Coordinate Geometry section in G.C.E. (A/L) is not at a satisfactory level. This could be due to many reasons such as lack of students' achievements, abilities, teachers' support and inappropriate teaching strategies, lack of students' interest, and other psychological aspects. This study investigates the productive teaching-learning methods to improve students' geometric skills and performance in Coordinate Geometry. The sample consisted of 102 students in G.C.E. (A/L) classes and 10 teachers in selected schools in Colombo and Gampaha districts in the Western Province in Sri Lanka. Initially, the sample was divided into two homogeneous groups based on the pretest marks. The treatment group ($n = 51$) received a constructivist workbook focusing on basics in Coordinate Geometry, and the control group ($n = 51$) received traditional instructions on the subject. In this mixed-methods study, pre-test, post-test, self-study pack, questionnaire, and semi-structured interview schedules and google forms were used to collect data. Quantitative data were analyzed using SPSS 20.0, whereas qualitative data were analyzed using thematic analysis. The results of the post-test depicted a significant difference between the experimental group and the control group ($p < 0.001$). The experimental group quickly grasped the concepts with the help of the workbook and it motivated students. Therefore, this teaching-learning method could build up students' self-concepts that directly affect students' performance in Coordinate Geometry. Thus, it is suggested to practice such methods to enhance students' meaningful learning in Coordinate Geometry in schools.

Keywords: Constructivist workbook, Coordinate Geometry, Meaningful learning

**IMPROVING ACHIEVEMENT LEVEL OF MATHEMATICS AMONG GRADE
EIGHT STUDENTS IN GANEWATTA EDUCATIONAL DIVISION:
AN ACTION RESEARCH**

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The aim of this action research was to increase the achievement level of Mathematics by 10% among Grade 8 students in Ganewatta Educational Division (ED) by creating a learner-friendly environment. In Ganewatte ED, there were nine schools out of 26 which had secondary grades, Grades 6 to 11. As an In Service Advisor (ISA) for Mathematics, a significant decline was observed in Mathematics achievement level among students in Grade 8 in comparison to Grades 6 and 7. This decline continues up to Grade 9 and Grade 10, before improving again in Grade 11 due to remedial programmes. The Grade-wise term test marks of first and second terms were analyzed to identify the issues. Other relevant information was gathered from Mathematics teachers at ED level quality circle meetings. Accordingly, two interventions were implemented during a period of three months. Following a thorough reflection on the problem, Mathematics wallpaper was identified as the first intervention. This was implemented in seven schools (two omitted due to some identified reasons). Since the wallpaper has motivated students' interest in Mathematics, the second intervention was introduced, which includes a Mathematics procession (*Perahera*) followed by a test and a seminar. Due to time constraints, the second intervention was held in three schools. The test results showed a remarkable increase in marks than the previous term. The number of students who gained marks below 40 was 63.3 % following the first intervention and it dropped to 36 % after the second intervention. Hence, the results of interventions demonstrate remarkable enhancement to arouse students' interest in Mathematics.

Keywords: Mathematics, Teacher-intervention, Wallpaper

ENHANCING MULTIPLICATION SKILLS WITH REGROUPING: AN ACTION RESEARCH WITH GRADE 8 BILINGUAL STUDENTS

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Multiplication is one of the four basic mathematical operations that needs in our day-to-day lives. The main target of this action research was to develop skills of multiplying two two-digit positive integers with regrouping. Five students, namely A, B, C, D and E, were randomly selected to conduct this action research from a class of 40 students. We used the qualitative method to analyze data. From the three pre-tests conducted, students A and C showed no proper knowledge on the place values while B and C were unable to memorize the multiplication table. The students D and E were unable to multiply the integers with regrouping. This action research was conducted in three cycles. In the first cycle, let the students to memorize the multiplication table and identify patterns in the multiplication table. After the first intervention, the students developed self-esteem and as a result they engage in lessons actively and enthusiastically. The second cycle was planned after considering the output of the first cycle with two group activities to measure their achievements. In the third cycle, two techniques were introduced that can be used when multiplying two two-digit numbers (using fingers and Criss cross multiplication method). At the end of the third cycle, their performance was assessed using the same pre-test paper. The observations revealed students' interest in engaging in group activities of learning mathematics. Students D and E overcame their problems and improved their achievement levels. Hence, current practices in facilitating students through diagnosing their weaknesses are to be promoted for enhancing students' performance in mathematics.

Keywords: Integers, Mathematics Education, Multiplication, Regrouping

**PROBLEMS ENCOUNTERED BY TEACHERS AND STUDENTS IN TEACHING
AND LEARNING MATHEMATICAL CONCEPTS RELATED IN
ALGEBRA AT G.C.E. (O/L)**

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Mathematics enhances critical thinking of individuals. Among the six fundamental themes in Mathematics, Algebra is weighted as the second in G.C.E. (O/L) Mathematics syllabus. Therefore, it is essential for students to gain proper understanding on Algebra. However, the reviews of G.C.E. (O/L) Mathematics evaluation reports of 2016, 2017, and 2018 highlight students' limited understanding on Algebra. This action-based research was conducted to investigate, (i) Algebraic concepts on which G.C.E. (O/L) Mathematics question paper I is prepared, (ii) students' competency levels on Algebraic concepts, and (iii) problems encountered by teachers and students in teaching-learning process. The sample consisted of 60 students (32 males and 28 females) of Grade 11 and 16 teachers from a national school. The data were collected through interviews, evaluating examination papers, and administering two separate questionnaires for students and teachers. G.C.E. (O/L) Mathematics evaluation reports were also studied. Data were analyzed using Microsoft Excel software. Based on the results, the lack of understanding on theory (70%), basic Algebraic concepts (60%), weakness of resolving equations (60%), limited understanding on simple linear equations (60%), weakness of separating factors (55%), limited understanding on the multiplication table (55%), and weakness of solving Algebraic fractions (45%) are identified as problems encountered by students. The lack of attractive teaching-learning strategies in the classroom is identified as one of the reasons for the above mentioned difficulties. This study also revealed the lack of students' critical thinking ability due to existing substandard teaching-learning methods. The lack of opportunities available for teachers to learn new teaching methodologies was also highlighted. About 60% of teachers showed their interest in enhancing their knowledge and skills on the subject. Teachers also expressed their difficulties of using more attractive practical activities in Mathematics due to lack of time and the heavy content of the syllabus. Hence, further research has to be conducted to identify the remedial measures to enhance students' meaningful learning.

Keywords: Algebra, Essential learning concepts, Mathematics

TEACHING MATHEMATICS BY PROSPECTIVE TEACHERS IN COLLEGES OF EDUCATION DURING INTERNSHIPS: AN ACTION RESEARCH

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Although Mathematics is an important core subject in the national curriculum, nearly 30% of the total candidates fail the subject every year at the G.C.E. (O/L) Examination. Constructive measures have been taken in order to make learning Mathematics an easy and interesting experience. The aim of this research is to examine the success of action research conducted by prospective teachers in enhancing students' Mathematics learning. The objectives were: (a) to study the effectiveness of the college curriculum to study the themes selected by the prospective teachers on action research; (b) to identify the challenges faced by prospective teachers in performing action research; and (c) to study the effectiveness of implemented action research to make suggestions in enhancing students' Mathematics learning. Quantitative dominant mix methods approach was used in the study. The sample consisted of randomly selected 35 prospective teachers and three lecturers. Data gathering instruments were interview schedules. Coding and categorizing were used in analyzing qualitative data. Quantitative data were analyzed using MS-Excel. Seven action research reports were analyzed with the help of an observation schedule and presented quantitatively. According to the findings, prospective teachers' knowledge about action research was adequate (76%). Moreover, selecting themes relating to basic mathematical operations was 38%. However, as teachers are heavily involved in teaching-learning process, 80% of prospective teachers felt that action research as a challenge. Only 75% students were involved in action research. Though the prospective teachers were competent enough in conducting action research, they found it as a challenge due to time constraints. Hence, the schools should play a supportive role in helping prospective teachers in conducting action research during their internship.

Keywords: Action research, Mixed methods, Prospective teachers, Quantitative aspects, Random sample

INVESTIGATING THE EFFECTIVENESS OF APPLYING FLIPPED CLASSROOM METHOD IN TEACHING MATHEMATICS

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Mathematics is a fascinating field that develops person's Higher Order Thinking (HOT) skills such as reasoning, creative thinking, and problem-solving. Mathematics is also important for a person's success of daily life. Therefore, Mathematics is taught as a compulsory subject in the school curriculum. However, it can be seen that the results of Mathematics at the G.C.E. (O/L) are not up to the expected level due to many reasons including students' inability to grasp the concepts that the teacher delivered as well as students' inability to reflect upon those concepts and to explore new knowledge. Therefore, the aim of the research is to explore the potential of flipped classroom in increasing the achievement level of Mathematics among students during the period of transition from concrete to abstract thinking by providing them self-directed learning opportunities. The research was conducted using experimental method with 40 students selected through convenience sampling technique from Grade 6 in Kegalle Educational Zone. Initially, the first six units in the Grade 6 syllabus were used and one class was taken as the control group and another as the experimental group. For the remaining six units, the control and experimental groups were swapped for data triangulation. A localized flipped version based on student textbook and teacher guide was used and intervention was implemented through activity sheets. Grade 5 scholarship marks of students were analyzed and pre- and post-tests were conducted. Observations, photographs, and assessments were used as data collection tools and techniques. Qualitative data was analyzed using appropriate methods while quantitative data by MS-Excel. The findings revealed that students in flipped classrooms gained relatively high scores. Hence, the results demonstrate that students should focus on self-studies which also help in time management in the learning-teaching process.

Keywords: Activity sheets, Control group, Experimental group, Flipped classroom, Post-test, Pre-test

**EXPLORING EASY TECHNIQUES FOR TEACHING AND LEARNING
CIRCULAR THEOREMS IN GEOMETRY THROUGH INDUCTIVE LOGIC
APPROACH FOR THE STUDENTS IN GRADES 10 AND 11**

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In 2018, the pass rate of Mathematics at the General Certificate Examination (G.C.E.) Ordinary Level (O/L) was 67%. There is a possibility of achieving approximately 32% of marks for the Mathematics question paper at G.C.E. (O/L) by answering the Geometry related questions in the paper. Out of these marks 12% of marks are allocated for the circular theorem related questions. Therefore, it is obvious that there is a significant impact on low results in Mathematics if students do not have adequate knowledge in this area. Mathematics is one of the core subjects at G.C.E. (O/L) and also has high expectations on academic and professional life of a person. Application of circular theorem also can be seen in many occasions related to number of professions. Hence, the aim of this mixed methods research was to explore easy techniques for learning and teaching of circular theorems related content in Geometry through inductive logical approach for students in Grade 10 and Grade 11 in order to improve their achievement levels in Mathematics. The convenient sample is consisted of 30 students in Grade 11 taken from two schools belonged to the Laggala Pallegama Education division in the Wilgamuwa Education Zone in Matale district, Central Province. Diagnostic test (pre-test), post-test, self-concept tests, focus group discussions with students, questionnaires, and students' exercises were used in the data collection process. Based on the diagnostic test results, appropriate techniques and learning teaching aids were developed and practiced in the teaching and learning process for concept building, establishing and providing feedback. The post-test results revealed a considerable decrease in the number of students belongs to the 15-20 class interval compared to the pre-test. It is also found that the techniques used in the teaching and learning process remarkably assisted to establish relevant geometry concepts among students in the sample.

Keywords: Circular theorem, Geometry, Inductive logical approach, Mixed methods

**USE OF CONCRETE PICTORIAL APPROACH (CPA) FOR TEACHING
MATHEMATICS: A CASE STUDY IN KULIYAPITIYA EDUCATIONAL ZONE,
SRI LANKA**

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Mathematics consists of many abstract concepts which are difficult to understand by the students whose cognitive development is not built up enough for the required level. The Concrete Pictorial Approach (CPA) that supports students' cognitive development has been introduced by the Ministry of Education to Sri Lanka. CPA consists of three stages; teaching Mathematical concepts through physical objects to guide in solving problems, use diagrams to solve these problems, and allow students to solve Mathematical problems without using physical objects or diagrams. The aim of this study was to examine whether the CPA influences the meaningful learning of Mathematical concepts introduced into Sri Lankan school curriculum of mathematics. The objectives were to: (a) identify teaching methods used by teachers to teach Mathematics from Grade 3 to Grade 7; (b) explore how they use the CPA approach in the learning-teaching process; (c) investigate teachers' views on the use of the CPA approach in the teaching and learning process of mathematics; and (d) find out barriers for the teachers on implementing CPA in classrooms. The sample was selected purposively and consisted of 10 teachers each from Type 2, Type 3 and 1C schools who taught for Grade 3. An open-ended questionnaire and an interview schedule were used to collect data. MS-Excel was used in analyzing data from the questionnaire and the Thematic Analysis was used to analyze interview data. It was revealed that 63% of the teacher sample used usual teaching methods while the rest were using the CPA approach in classrooms but not knowing that they were using the CPA approach. Further, 47% of them reported that the time allocated in the syllabus was not sufficient to carry out CPA and 77% of teachers believed that CPA approach was impractical to use in the classroom as the current education system is competitive and examination-oriented. However, based on the findings of this study it reveals that the use of CPA is possible if the amount of subject content could be reduced in Grade 3 to Grade 7 which eventually help students to easily understand Mathematical concepts. Moreover, it is better if there is a system to measure students' cognitive skills as well as psychological skills in the classroom to offer autonomy support for students to enhance their achievement levels in Mathematics.

Keywords: Concrete Pictorial Approach, Mathematics teaching, Thematic Analysis

**PROBLEMS AND SOLUTIONS FOR LOW ACHIEVEMENT LEVELS IN
MATHEMATICS: ACTION RESEARCH WITH SPECIAL REFERENCE TO
WILGAMUWA EDUCATION ZONE**

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The second highest fail rate in Mathematics at the G.C.E. (O/L) Examination is reported from the Wilgamuwa Education Zone in the Central Province of Sri Lanka. Accordingly, the main objective of this research was to identify the reasons for students' low achievement in Mathematics and to increase Mathematics achievement level of Grade 11 students by using appropriate remedies. Out of the 15 schools offering G.C.E. (O/L) in the Wilgamuwa Educational Zone, required data was gathered from 100 students of Grade 11 from a Type 1C school. The term test marks were analyzed under four class intervals and basic discussions were held to identify the reasons for low achievement levels. Discussions revealed four issues related to the low performance of students. Answer scripts of all the students were checked to understand how they answered the questions under six content themes. Theme-based worksheets with basic concepts were given as the first intervention. It was observed that most of the students understood the basic concepts and became self-motivated. Therefore, as the second intervention, students were given theme-wise simple problems to solve, which they have done easily with the knowledge of basic concepts. As the third intervention, they were given complex problems to solve in order to allow them to demonstrate their confidence in solving problems from simple to complex. As the final intervention, examination-based prototype questions were given. The findings revealed that the percentage of students in the 0 - 60 interval had dropped from 77% to 38% after the interventions. The number of students answering questions and the accuracy under each theme had also increased. Thus, the study concludes that these interventions have increased the confidence of students by overcoming their fear on Mathematics. The findings suggested that it is possible to increase the passing rate in Mathematics in areas such as Wilgamuwa after identifying strengths and weaknesses of students and implement interventions accordingly from early grades of secondary level.

Keywords: Class intervals, Content themes, Interventions, Worksheets

STRATEGIES TO ENHANCE THE ACHIEVEMENT LEVEL IN LOW PERFORMING MATHEMATICAL THEMES IN MATHEMATICS PAPER I IN THE BASIC PIRIVENA FINAL EXAMINATION: A CASE STUDY IN KELANIYA AND NEGOMBO EDUCATIONAL ZONE

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Studying mathematics will develop intelligence and decision making ability of a person. Mathematics is an essential subject not only for school children but also for lay clergy students studying in more than 800 pirivenas across Sri Lanka. As the students in schools sit for the General Certificate Examination (G.C.E. O/ L), clergy students of Primary and Maha Pirivenas sit for the final examination of the Basic Pirivena. In that, it is compulsory to pass Mathematics to pursue higher education. Mathematics Paper-I at the Basic Piriven Final Examination consists of 20 short questions based on Essential Learning Concepts (ELCs). Though it is not difficult compared to the Mathematics Paper II, a majority of Piriven students fail in Mathematics Paper I. The aim of this study was to identify Mathematics themes that students find difficult to answer in Mathematics Paper I and to formulate strategies to minimize those weaknesses. In this mixed methods research, the convenient sample included 60 lay clergy students from ten Pirivenas in the Western Province of Sri Lanka who are planning to sit the final examination of Piriven in 2021. A model question paper and interviews were used as research tools. The marks obtained for the question paper were analyzed. The students scored approximately 33%, 27 %, 33%, 17%, 75% and 50% for different themes *viz.*, Number, Measurement, Algebra, Geometry, Set theory and Probability and Statistics, respectively. The results revealed that students performed poorly especially in Geometry and Measurement. As such, it is suggested the importance of using appropriate remedial measures to enhance the achievement level of students.

Keywords: Essential Learning Concepts, Mathematic themes, Mixed methods, Model paper

EFFECT OF TEACHING METHODS IN STUDENTS' ACHIEVEMENTS IN MATHEMATICS

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As a compulsory subject, Mathematics occupies a significant place in the primary and secondary school curriculum in Sri Lanka. The Government has taken various measures to improve the quality of teaching and learning processes in Mathematics. However, according to the past statistics, the students' passing rate of Mathematics in G.C.E. (O/L) Examination is rather low. Since 2015, the passing rate has increased only by 3%. The records produced by the Department of Examinations show that nearly 25% of G.C.E. (O/L) students fail Mathematics every year. According to the literature, teaching methods have a direct impact on students' performance in Mathematics. The aim of this study was to critically examine how teaching methods affect students' achievement levels in Mathematics and to suggest necessary action to address those shortcomings. The convenient sample included 20 Sinhala medium Grade 7 Mathematics teachers and 100 Sinhala medium Grade 7 students in ten schools from five educational zones in the Western Province of Sri Lanka. Mixed methods approach was used. Questionnaires, checklists, and documents (exercise books, mark sheets and teacher instructional manuals) were used as research instruments. Data were analysed using MS Excel. The findings revealed that the number of students obtained marks in the range of 1 to 20 has increased from 5% (in the first term) to 8% (in the third term) while, the the number of students obtained marks in the range of 81 to 100 has decreased from 28% to 13%, respectively. According to the findings, pedagogical practices followed by teachers were not satisfied. The lack of engagement in pedagogical practices were reflected in organizing lessons attractively (20%), evaluating and giving feedback (0.28%), examining student exercise books (34%), managing time effectively (15%), following teacher instruction manuals (20%) and using various teaching methods (25%). It is also observed that there is no continuous supervision of the teaching process. Hence, it can be suggested that the level of students' achievement can be enhanced by improving the quality of teaching through mentoring and continuous monitoring. The findings of this research can be used as a valuable information during policy changes in the field of Education.

Keywords: Learning teaching methods, Mathematical achievement, Mathematics

**RELATIONSHIP BETWEEN STUDENTS' ATTITUDE AND THEIR
PERFORMANCE IN MATHEMATICS**

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It is a common belief that the student's attitude towards mathematics play a prominent role in one's achievement. Thus, the aim of this research is to investigate the relationship between students' attitudes towards mathematics and their overall performance. All Grade 10 students of five schools from Kantale Educational Zone were purposively selected for the study, totaling 261 students. Term test marks obtained by the students for Mathematics subject were used to evaluate their performance, while a questionnaire was prepared to measure their attitudes. This four-dimensional (Anxiety, Motivation, Value, and Confidence) Likert scaled questionnaire consists of 21 statements (Cronbach alpha= 0.810). Data were analyzed descriptively and inferentially using SPSS software. An independent sample *t* –test, correlation, two-way ANOVA and multiple regression were utilized for further analyses. Independent sample *t* –test revealed that female students performed better than the male students ($p < 0.001$, difference= 7.312). Correlation analysis indicated that all subscales related to students' attitudes were significantly correlated with their achievement ($p < 0.001$). Results also revealed no significant difference between student motivation and gender, whereas statistically significant differences were noted with the remaining subscales ($p < 0.05$). Multiple regression analysis revealed that the overall model was significant ($R^2 = 0.465$, $p < 0.001$). The results confirmed a strong relationship between students' attitudes and their achievement levels in Mathematics. Therefore, the study suggests that teachers should incorporate new teaching methods to sustain their interests and minimize their anxiety towards learning Mathematics.

Keywords: Achievement, Anxiety, Attitude, Motivation

**TEACHER PERCEPTIONS OF TEACHING MATHEMATICS THROUGH
ONLINE METHODS: A CASE STUDY IN WESTERN PROVINCE**

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Mathematics is the science that contributes to our day-to-day lives. Thus, Mathematics has an important place in education system. In the school curriculum, Mathematics has been taught in classroom settings using teacher-student interactions. However, once the schools were forced to close due to the Covid-19 pandemic since 2019, the teachers were resorted to conduct the learning and teaching process online. Previous studies have shown that there are barriers that need to be addressed when teaching Mathematics online. This study was conducted in two education zones in the Western Province, Sri Lanka. A sample of 150 teachers involved in online Mathematics teaching for Grade 6 to Grade 11 was selected using convenience sampling. Data was collected through an online questionnaire to identify the issues in teaching Mathematics online and to find out softwares that they used in the teaching-learning process in Mathematics to explore their limitations. Open-ended questions were analyzed by thematic analysis, while the close-ended scaled data analyzed through SPSS 20. The results revealed that online teaching of Mathematics seems to be the only alternative as the schools are closed due to the pandemic. Teachers also had to choose a software that is available for majority of students. The teachers are facing many challenges related to software, internet connection, lack of technical knowledge, money constraints, and lack of support from the school administration during online teaching. It was also noted that online means are not suitable for teaching certain topics in Mathematics. Despite the measures taken to resolve some issues on a temporary basis, a majority of students still face difficulties due to lack of needed facilities. The study highlights the necessity of improving the infrastructure for online teaching-learning process at the national level and introducing programmes to update the knowledge of teachers to enhance the meaningful online learning.

Keywords: Covid-19 pandemic, Online education

RELATIONSHIP BETWEEN MATHEMATICS SELF CONCEPTS AND QUALITATIVE ACHIEVEMENT OF SECONDARY LEVEL STUDENTS

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Mathematics is a compulsory subject for G.C.E. (O/L) students to qualify for G.C.E. (A/L) in Sri Lanka. Among the factors which affect students' achievements in Mathematics, self concepts can have a powerful influence over behavior, thus self concepts can make a difference in students' performance. Thus, the aim of this mixed methods study is to find out the relationship between the secondary level students' qualitative achievements in Mathematics and self concepts in Mathematics. Thirty students from 6 different secondary schools in Walapane Educational Zone were selected as the study sample. The standard test paper prepared by the Provincial Examination Department was used to measure the students' achievements. A self constructed questionnaire was used to find out the students' self concept towards Mathematics Education. The data was analyzed using MS-Excel. The study showed that the secondary level students have a considerably high self-concept in Mathematics (Mean = 81.75; SD = 11.73) and moderate level of achievement (mean = 45.9; SD = 21.2). Meanwhile, there was a slightly low qualitative achievement (over 55 marks = 36.6%). Furthermore, it was revealed a low positive relationship between the achievement and self-concepts in Mathematics (Pearson's correlation coefficient = 0.25). Nevertheless, the students who have high self concepts in Mathematics show higher achievement. Thus, it is important to consider students' self-concepts in Mathematics to enhance the qualitative achievement in Mathematics among secondary level students.

Keywords: Qualitative achievement, Self-concept

EFFECTIVENESS OF USING NEW SOFTWARE TO TEACH GEOMETRIC CONSTRUCTIONS FOR GRADE 9 STUDENTS: A CASE STUDY FROM HINGURAKGODA EDUCATIONAL ZONE IN NORTH CENTRAL PROVINCE, SRI LANKA

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Mathematics is treated as a universal language for rational and logical thinking. Therefore, students should be more pedagogically directed to engage in Mathematics and self-studies. In the secondary education, Mathematics is learnt under six main themes including Geometry. Under Geometry, one of the skills taught is the geometric constructions which promotes creativity and application skills among students. At the G.C.E. (O/L), 10 marks are allocated for the geometric question in Part B of the Mathematics Paper II. Students who have learnt basic geometric constructions are able to answer this question and to select appropriate constructions in each step. Hence, aim of this research was to identify the difficulties that students are facing in answering geometry-related questions and to develop a suitable software to resolve them. The mixed method was used for data collection. The marks obtained by the students at G.C.E. (O/L) examination from 2015 to 2018 for the geometric construction question were analyzed. Analysis revealed that 89% – 93 % of students had selected the geometric construction question and facility index from 53% to 65%. From that, 45% -69% students gained 0 - 5 marks (out of 10) during last four years. Data collection was done through questionnaires for students and teachers, and interviews with students. When the data were analyzed, students' average mark was in the range of 4.0-5.4 out of 25 for the 11 geometric construction questions. The findings of the interviews indicated the lack of knowledge on geometric constructions among students possibly due to poor standards of existing methods. Findings revealed that existing methods of teaching was insufficient to understand the steps in final answer/ constructions. Thus, a software was developed that can be used to teach each step taken in geometric constructions as well as in self-studies of Geometry. This software will be implemented and will check for its effectiveness as a next step of this research.

Keywords: Basic constructions, Facility index, Geometry, Pedagogical knowledge

**CHALLENGES IN STUDYING ALGEBRAIC EXPRESSIONS OF GRADE 3 IN
PIRIVENA EDUCATION**

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Mathematics enhances critical thinking ability of the learner, thus the main goal of Mathematics Education is to produce an individual with rational and critical thinking. It is expected that the learner will apply the skills and critical thinking ability gained by learning Mathematics in their daily lives for problem solving and other situations. In the Secondary Education, Mathematics is categorized under six themes and Algebra is one of them that improves the learners' critical thinking ability. Algebra question papers 1 and II at the Pirivena Final Examination assess the learner's algebraic skills. Though teaching Algebra commences from Pirivena Grade 1, it is observed a low achievement level among students in Pirivena Grade 3 in topics such as simplifying algebraic expressions with similar terms, minimizing algebraic expressions by removing brackets, and substituting algebraic expressions. Therefore, the aim of this mixed methods research was to develop and practice a teaching method by using technology such as video clips etc. to visualize the process of learning algebraic expressions to increase the interest and motivation among the learners. The sample of this study consists of 90 students in Grade 3 from 13 Pirivenas in Ampara Educational Zone. Focus group discussion schedule, questionnaires, pre-test, and a post-test were used as data gathering instruments. Data analysis was done using MS Excel. Findings revealed that at the pre-test, about 20 % of participants were unable to simplify algebraic expressions with similar terms, while 45% was unable to minimize algebraic expressions by removing brackets and 30% failed in substitution of algebraic expressions due to their inability to apply sequential order in making sums. However, at the post-test, almost 80% of students were able to do all three components. Hence, it is suggested that technology-based visuals can be used in teaching relevant components in Algebra as a way to improve the interest and motivation of the learner.

Keywords: Algebra, Algebraic expressions, Critical thinking, Mixed methods, Questionnaires

**RELATIONS OF STUDENT MOTIVATION AND SELF-CONCEPT WITH
LEVEL OF ACHIEVEMENT IN MATHEMATICS OF PRE-PRIMARY LEVEL IN
PIRIVENAS: A CASE STUDY IN PIRIVENAS IN POLONNARUWA
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Mathematics has a special place in school education, which aims to endow children with a balanced personality to face challenges in a constantly changing world. Bernard Foe claims that Mathematics is a powerful tool for rational thinking. Mathematics is considered as a core subject in the school curriculum. However, the students who fail Mathematics in the G.C.E. (O/L) examination may not have the opportunity to pursue higher education. Therefore, the main objective of this mixed methods study is to investigate the relationships of student motivation and self-concept with Mathematics achievement level of pre-Primary students in Pirivena to make necessary suggestions to increase the level of achievement in Mathematics. In the study, the relationship between student motivation and achievement level, as well as self-concept and achievement level were explored in order to identify corrective strategies to increase the level of achievement in Mathematics. Term test marks and questionnaires were used as data collection tools in this study. Convenience sample of 50 students and ten teachers from ten Pirivenas in Hingurakgoda, Tamankaduwa and Dimbulagala Educational Zones of the Polonnaruwa District in the North Central Province were taken as the sample. Findings revealed that even if students' self-concept level is high, their achievement level is low. Outcomes of the interviews revealed that the teaching process influenced achievement levels of Mathematics among students. Therefore, the use of different strategies such as interesting teaching methods, different motivational techniques, games, stories can enhance achievement levels in Mathematics. In addition of using appropriate teaching strategies, introducing short arithmetic techniques to enhance memorizing multiplication tables, and assigning activity-based homework can be considered as effective ways to improve student performance in Mathematics.

Keywords: Mix methods, Motivation, Self-concept, Strategies

**IMPROVING THE ABILITY OF ADDITION OF INTEGERS AMONG
GRADE 10 STUDENTS**

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Mathematics is important in day-to-day activities in life including decision making and problem solving etc. Mathematics also supports to develop higher order thinking skills of a person. Available data in the evaluation reports published by the Department of Examinations, highlights the areas that students perform poorly in Mathematics. Based on the information of those reports, addition of integers which represents Algebra and Numbers was identified as the area that needs attention. Therefore, the aim of this action research was to develop skills on addition of integers in a sample of 35 students in Grade 10 from a school in the Galle Educational Zone in the Southern Province of Sri Lanka. Initially, the problem was identified through observations and then confirmed by a diagnostic test. As the first intervention, six steps of activities of the teaching-learning process of addition integers were introduced through oral and written tests. Other interventions were included classroom group activities, individual activities, game activities and the use of Nomogram mathematics tool, respectively based on six steps of activities of addition of integers. The Nomogram tool was used only for checking the answers. A pretest, posttest, term test mark sheets of students, activity sheets, informal discussions and reflective notes were used as data collection instruments. Data was analyzed using MS-Excel software. Findings revealed that 13 students out of 35 (37%) were belonged to the category of below 1 mark at the pretest. However, after all interventions, 25 (71.4%) students obtained higher marks (>4 marks category) in the post test. It was also observed that students engaged actively and collaboratively in learning Mathematics due to the integration of different activities. Hence, the integration of appropriate activities in teaching Mathematics provides a meaningful learning experience.

Keywords: Action research, Addition of integers, Nomogram tool

LEARNING MATHEMATICS THROUGH ONLINE METHODS

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The aim of this study is to explore factors affecting student satisfaction in online learning activities of secondary schools in the Kelaniya Educational Zone in the Gampaha District, Sri Lanka. The factors examined in this research were interaction between teachers and students, and nature of teaching; methods and learning strategies used by teachers for online learning of Mathematics. This is a mixed method study. Though the online learning of mathematics has been implemented in many schools since the Covid-19 pandemic, the sample was randomly selected from schools in Kelaniya Education Zone. The sample consists of 200 students from four schools in the Kelaniya Educational Zone. A google form with short multiple-choice questions were used to collect data as the research instrument. Data analysis was done using Microsoft Excel and it revealed that students prefer solving problems in mathematics with the help of their peers rather than doing it individually at home. It was also noted that interactions between teacher and students were at a minimum level through online learning. Thus, the study suggests that more attention should be given to activity-based assignments to gain first-hand experience related to the subject content. It will pave the way for students to learn Mathematics in an interesting manner, especially when learning Geometrical concepts. In order to keep students' interest on online learning, it is advisable to carry out online teaching-learning even once the schools are reopened, at least once a week, to develop students' skills in online teaching and learning and to maintain their interests.

Keywords: Online learning, Online teaching

TEACHING-LEARNING PROCESS IN COMBINED MATHEMATICS: A CASE STUDY ON TWELVETH GRADE IN KANDY EDUCATIONAL ZONE

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Mathematics plays an important role in Education. It is also an important subject in the senior secondary school curriculum in Sri Lanka as Combined Mathematics (CM) in G.C.E. (A/L) under the Physical Science stream. Evaluation reports of the Department of Examinations indicate that the achievement level of students in CM is low compared to Physics and Chemistry in G.C.E. (A/L). Therefore, the main purpose of this research is to explore teachers' and students' perceptions on teaching-learning process of CM in order to investigate the strengths and issues in the process and provide constructive suggestions. Multiple method approach was used in this research with purposive sample of 157 students in the Kandy Educational Zone. Data collection was done by using semi-structured interview schedules and questionnaires. Quantitative data was analyzed by using MS Excel and qualitative data by thematic analysis and grounded theory. The findings revealed that 94% of the students realized that CM as a challenging subject, which needs deep understanding and requires more dedication than Mathematics in G.C.E. (O/L). Despite its difficulty, 87% of students stated that CM is their most favorite subject. Regarding the way of learning of CM, 46% of students stated that they learn the subject at schools with the help of well-explained theory and discussions of questions from past papers. Accordingly, 37% of students achieved it with the help of theory and discussions of past papers while 18% of students learn the subject through informal ways. Majority of teachers (84%) in the sample believed that the lecture method is the best method according to their experience. The findings suggested that it is essential to improve the quality of teaching among CM teachers. In order to achieve this, conducting workshops would be beneficial for newly appointed teachers to improve their quality of teaching.

Keywords: Combined Mathematics, Teaching learning process

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